Table 1: Important Maclaurin Series and their Radii of Convergence

$\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$	$R = 1$
$e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!} = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots$	$R = \infty$
$\sin x = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{(2n+1)!} = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \cdots$	$R = \infty$
$\cos x = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n}}{(2n)!} = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \cdots$	$R = \infty$
$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{2n+1} = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \cdots$	$R = 1$
$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \dfrac{x^n}{n} = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \cdots$	$R = 1$
$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \dfrac{k(k-1)}{2!}x^2 + \dfrac{k(k-1)(k-2)}{3!}x^3 + \cdots$	$R = 1$

Alternating Series Estimation Theorem

If $S := \sum_{k=r}^{\infty} (-1)^k b_k$, where $b_k > 0$, is the sum of an alternating series that satisfies

$$\text{(i) } b_{k+1} \leq b_k \qquad \text{and} \qquad \text{(ii) } \lim_{k\to\infty} b_k = 0,$$

then $|R_N| = |S - S_N| \leq b_{N+1}$, where $S_N := \sum_{k=r}^{N} (-1)^k b_k$.

Taylor's Inequality

If $|f^{n+1}(x)| \leq M$ for $|x - a| \leq d$, then the remainder $R_n(x)$ of the Taylor series satisfies the inequality

$$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1} \text{ for } |x-a| \leq d$$

Volume

The solid obtained by rotating the region under the curve $y = f(x)$ from a to b about the x-axis has volume

$$\int_a^b \pi(\text{radius})^2 \text{ dx} = \int_a^b \pi(\text{f(x)})^2 \text{ dx}$$

Useful trig facts.

$$\sin^2\theta + \cos^2\theta = 1, \quad \tan^2\theta + 1 = \sec^2\theta$$

$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta), \quad \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta, \quad \sin 2\theta = 2\sin\theta\cos\theta$$

$$\sin\frac{\pi}{6} = \frac{1}{2}, \quad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2},$$
$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}, \quad \cos\frac{\pi}{3} = \frac{1}{2},$$
$$\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

Some derivatives.

$$\frac{d}{dx}b^x = \ln(b)b^x$$

$$\frac{d}{dx}\sin(x) = \cos(x) \qquad \frac{d}{dx}\cos(x) = -\sin(x) \qquad \frac{d}{dx}\tan(x) = (\sec(x))^2$$

$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x) \qquad \frac{d}{dx}\sec(x) = \sec(x)\tan(x) \qquad \frac{d}{dx}\cot(x) = -(\csc(x))^2$$

Fundamental Theorem of Calculus, part I.

If f is continuous on $[a,b]$, then function g defined as

$$g(x) = \int_a^x f(t)\,dt, \quad a \le x \le b$$

satisfies $g'(x) = f(x)$.

Fundamental Theorem of Calculus, part II.

If f is continuous on $[a,b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where F is any anti-derivative of f (ie. F is any function such that $F' = f$).

Integration by parts fomula.

$$\int u\,dv = uv - \int v\,du$$