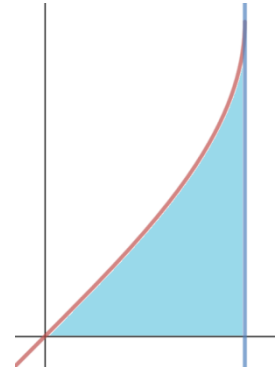


MATH 1152Q Exam 1 Summary

[Sec 7.1-7.3] Techniques of Integration

【7.1】 Integration by Parts

1. Evaluate $\int x \tan^2 x \, dx$.
2. Evaluate $\int x \arctan x^2 \, dx$.
3. Evaluate $\int x^2 (\ln x)^2 \, dx$.
4. Evaluate $\int \sin(\ln x) \, dx$.
5. Consider the graph of the function $f(x) = \sin^{-1} x$. Let R be the region bounded by $y = f(x)$ and x -axis on the interval $[0, 1]$. Evaluate the **area** of R . Check that your answer is positive.

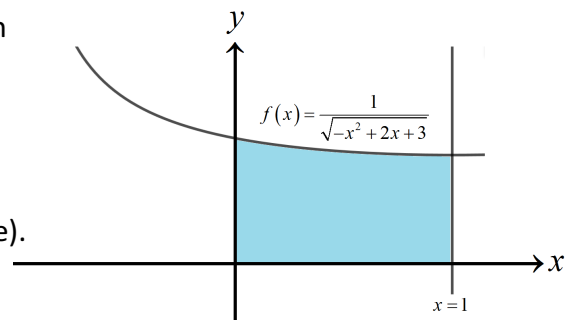


【7.2】 Trigonometric Integrals (trig identities will be provided)

1. Evaluate $\int \frac{\cos^2(\arctan x)}{1+x^2} \, dx$. (Hint: I prefer drawing a triangle for this one)
2. Evaluate $\int \sin^3 x \cos^4 x \, dx$.
3. Evaluate $\int \sin^4 x \cos^3 x \, dx$.
4. Evaluate $\int \tan^3 x \cdot \sec^2 x \, dx$.
5. Evaluate $\int \tan^2 x \cdot \sec x \, dx$.
6. Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{1 - \cos 2x} \, dx$.
7. Evaluate $\int \frac{1 - \tan^2 x}{1 + \tan^2 x} \, dx$.

【7.3】 Trigonometric Substitution (trig identities and table of substitution will be provided)

1. Let R be the region bounded by the function $f(x) = \frac{1}{\sqrt{-x^2 + 2x + 3}}$ and x -axis on the interval $[0, 1]$. Compute the area of R .
(Sanity check: your answer should be positive).



2. Evaluate $\int_1^e \frac{1}{y\sqrt{1+(\ln y)^2}} dy$. Sanity check: is your answer positive?

3. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{1+\sin^2 x}} dx$. Sanity check: is your answer positive?

4. Evaluate $\int \frac{1}{(x-1)\sqrt{x^2-2x}} dx$.

5. Evaluate $\int_{\sqrt{2}}^2 \frac{1}{x^3\sqrt{x^2-1}} dx$.

MATH 1152Q Exam 1 summary Answer

[Sec 7.1-7.3] Techniques of Integration

【7.1】 Integration by Parts

(1) $x \tan x + \ln |\cos x| - \frac{1}{2}x^2 + C$ (2) $\frac{x^2}{2} \arctan x^2 - \frac{1}{4} \ln(1+x^4) + C$

(3) $\frac{1}{3}x^3 (\ln x)^2 - \frac{2}{9}x^3 \ln x + \frac{2}{27}x^3 + C$

(4) $\frac{x}{2} [\sin(\ln x) - \cos(\ln x)] + C$ (5) $\frac{\pi}{2} - 1$ (check that this is actually positive!)

【7.2】 Trigonometric Integrals

(1) $\frac{1}{2} \arctan x + \frac{x}{2(1+x^2)} + C$ (2) $-\frac{1}{5} \cos^5 x + \frac{1}{7} \cos^7 x + C$ (3) $\frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + C$

(4) $\frac{1}{4} \tan^4 x + C$ (5) $\frac{1}{2} \sec x \tan x - \frac{1}{2} \ln |\sec x + \tan x| + C$, see

https://egunawan.github.io/fall17/notes/LA7_2part2key.pdf

(6) $\sqrt{2}$ (7) $\frac{1}{2} \sin 2x + C$

【7.3】 Trigonometric Substitution

(1) $\frac{\pi}{6}$ (b) $\frac{\pi}{4} \ln 3$ (2) $\ln(1+\sqrt{2})$ (3) $\ln(\sqrt{2}+1)$ (4) $\operatorname{arcsec}|x-1| + C$ (5) $\frac{\pi}{24} + \frac{\sqrt{3}}{8} - \frac{1}{4}$