University of Connecticut
Department of Mathematics

MATH 1152Q | SAMPLE EXAM 1 | FALL 2017

NAME: ____

- This sample is **a lot longer** than the actual test. The actual test will be about 8-9 pages with 1-4 questions on each page.
- From each page, choose a few questions that seem the most difficult.

Read This First !

- You will earn a small amount of bonus 'style points' for a legible, coherent, and non-ambiguous paper. In addition, your reader should not need to reread your solution several times to find a train of thought.
- Please read each question carefully. Show **ALL** work clearly in the space provided. In order to receive full credit on a problem, solution methods must be complete, logical and understandable.
- Answers must be clearly labeled in the spaces provided after each question. Please cross out or fully erase any work that you do not want graded. The point value of each question is indicated after its statement. No books or other references are permitted.
- All technology (phones, calculators) and books/ notes should be stored inside your bag.

1. For the following questions, circle TRUE or FALSE, and give a justification. Argue using facts, theorems or definitions from class. [6]

(a) If $\lim_{n\to\infty} a_n = 0$ then the series $\sum a_n$ converges. **T** **F**

Justification:

(b) If $a_n > 0$, $b_n > 0$ for all n, $\sum b_n$ diverges and $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$, then $\sum a_n$ diverges. **T** **F**

Justification:

(c) If $a_n > 0$ for all n & $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 0$, then $\sum a_n$ converges by the ratio test. **T** **F**

Justification:

(d) If $a_n > 0$, $b_n > 0$ for all n and $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$, then $\sum a_n$ is convergent. **T** **F**

Justification:

(e) The harmonic series $\sum \frac{1}{n}$ is convergent by the p-series test **T** **F**

Justification:

(f) We can use the ratio test *alone* to show the geometric series

$$\sum \frac{2^n}{3^n}$$

converges **T** **F**

Justification:

(g) We can use the p-series test *alone* to show the series

$$\sum \frac{2^n}{3^n}$$

converges **T** **F**

Justification:

(h) We can apply the monotonic sequence theorem to show that the geometric *sequence* $\left\{\dfrac{2^n}{3^n}\right\}_{n=1}^{\infty}$ is convergent **T** **F**

Justification:

(i) We can apply the monotonic sequence theorem to show that the harmonic sequence $\left\{\dfrac{1}{n}\right\}_{n=1}^{\infty}$ is convergent **T** **F**

Justification:

(j) We can apply the squeeze theorem to show that the alternating harmonic sequence $\left\{\dfrac{(-1)^n}{n}\right\}$ is convergent **T** **F**

Justification:

(k) The integration by parts formula can be derived immediately from the product rule.
T **F**

Justification:

(l) The integration by parts formula can be derived immediately from the chain rule **T** **F**

Justification:

(m) The integration by parts formula can be derived immediately from the u-substitution formula. **T** **F**

Justification:

(n) It is impossible for a subset of a line to have infinitely many points and have length zero. **T** **F**

Justification:

2. (a) State the contrapositive of the factual statement: "If the sequence $\{a_n\}$ is unbounded, then it is divergent". [2]

(b) Is the contrapositive statement you wrote in part (a) true? **(No justification necessary).** [2]

3. Answer the following on the line provided. No justification is necessary. [18]

(a) What is the 100th term of the sequence $\{2, 5, 8, 11, \ldots\}$?
(The terms 2 and 5 are the first and second term, respectively) ________________

(b) Find a formula for the general term a_n of the sequence $\{1, -\frac{2}{5}, \frac{3}{25}, -\frac{4}{125}, \frac{5}{625} \ldots\}$. Make sure to specify your starting value of n.

(c) Write the geometric series $4 + 2 + 1 + \frac{1}{2} \cdots$ in standard form. ________________
(summation notation $\sum_{n=\square}^{\infty}$)

(d) Find the 7th term a_7 in the recursive sequence $a_{n+1} = a_n + a_{n-1}$ ________________
when $a_1 = 2$ and $a_2 = 3$.

(e) We can use geometric series to compute

$$\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots.$$

What fraction is this equal to?

(f) We can use geometric series to compute $0.9999\ldots$. What fraction is this equal to?

(g) One of the two decimal expansions for a number is $2.44999999\ldots$. What is the other decimal expansion?

(h) Use geometric series to compute the fraction for $1.833333\ldots$

(i) Use geometric series to compute the fraction for $1.08333333333333333\ldots$

(j) Does the series $\displaystyle\sum_{n=1}^{\infty} -\ln\left(\frac{n}{2n+7}\right)$ converge or diverge? ______________

(k) Find the sum of the series $\displaystyle\sum_{k=0}^{\infty} 5\left(\frac{2}{3}\right)^k$. ______________

(l) Find the sum of the series $\displaystyle\sum_{n=2}^{\infty} 5\left(\frac{(-6)^{n-1}}{7^n}\right)$. ______________
You don't need to reduce your fraction.

(m) Find the sum of the series ______________

$$-5+3-\frac{9}{5}+\frac{27}{25}-\frac{81}{125}+\dots.$$

(n) Write an expression for the nth term in the sequence ______________
$\left\{\frac{1}{2},\frac{1}{6},\frac{1}{24},\frac{1}{120},\dots\right\}$. (The terms $\frac{1}{2}$ and $\frac{1}{6}$ are the first and second terms in the sequence)

(o) Write an equivalent series with index summation beginning at $n=0$. ______________
$\displaystyle\sum_{n=2}^{\infty}\frac{2^n}{(n-2)!}$

(p) For what values of k does the series $\displaystyle\sum \frac{5}{n^k}$ converge? ______________

4. The following questions ask you to determine the converge/divergence of a series. [5]

   (a) Use the **limit comparison test** to determine whether the series $\displaystyle\sum_{n=3}^{\infty} \frac{6}{n\sqrt{n^2-8}}$ converges or diverges.

   (b) Determine whether the series $\sum_{n=1}^{\infty} \dfrac{\sqrt{n^3+1}}{3n^3-4n+2}$ converges or diverges. Make sure to state which test/s you use and provide justifying computations and arguments.

   (c) Determine whether the series
   $$\sum_{n=1}^{\infty} \frac{n!}{7^n(n+8)!}$$
   converges or diverges. Make sure to state which test/s you use and provide justifying computations and arguments.

   (d) Determine whether the series
   $$\sum_{n=1}^{\infty} \frac{(n+8)!}{7^n\ \ n!}$$
   converges or diverges. Make sure to state which test/s you use and provide justifying computations and arguments.

   (e) Determine whether the series $\sum_{n=1}^{\infty} \ln\left(\dfrac{3n}{n+1}\right)$ converges or diverges. Make sure to state which test/s you use and provide justifying computations and arguments.

   (f) Determine whether the series $\sum_{n=1}^{\infty} \dfrac{(-1)^n}{5n+3}$ converges or diverges. Make sure to state which test/s you use and provide justifying computations and arguments.

5. Find the values of A so that the geometric series $\displaystyle\sum_{n=1}^{\infty} \frac{(A-3)^{n-1}}{3^{n-1}}$ is convergent. [8]

6. (a) Compute $\lim\limits_{n\to\infty}\left(1+\dfrac{1}{n}\right)^{3n}$. [5]

(b) Compute $\lim\limits_{n\to\infty}\left(1+\dfrac{1}{2n}\right)^{n}$.

(c) Compute $\lim\limits_{n\to\infty}\left(1+\dfrac{2}{n}\right)^{5n}$.

(d) Compute $\lim_{n\to\infty} n^2e^{-n}$.

(e) Compute $\lim_{n\to\infty} \dfrac{\ln n}{n}$.

(f) Compute $\lim_{n\to\infty} \dfrac{n\sin n}{n^2+1}$.

7. Consider the series $\sum a_n = \sum_{n=2}^{\infty} \frac{\cos(n\pi)}{n-1}$.

   (a) What are the first three terms in the series? ________________ [3]

   (b) Is the series convergent? You must justify. ________________ [5]

   (c) Is the series $\sum a_n = \sum_{n=2}^{\infty} \frac{|\cos(n\pi)|}{n-1}$ convergent? You must justify. ________________ [5]

8. (a) Evaluate $\lim_{n\to\infty} e^{-n}\sqrt{n}$. [5]

   (b) Determine whether

$$\sum_{n=0}^{\infty} e^{-n}\sqrt{n}$$

   converges or diverges.

   (c) Evaluate $\lim_{n\to\infty} \frac{(\ln(n))^2}{n^2}$.

   (d) Determine whether $\sum_{n=1}^{\infty} \frac{(\ln(n))^2}{n^2}$ converges or diverges.

   (e) Suppose $\sum_{n=1}^{\infty} a_n$ is a series with the property that

$$a_1 + a_2 + \cdots + a_n = 2 - 3(0.8)^n.$$

   State whether $\sum_{n=1}^{\infty} a_n$ converges or diverges. If it converges, find its sum.

9. Compute the following definite and indefinite integrals.

   (a) $\int_1^2 t^3 \ln(t) dt$ [5]

   (b) $\int \dfrac{2x-3}{8+x^2}$ dx [5]

   (c) $\int \dfrac{\sin(\ln(x))}{x}$ dx [5]

   (d) $\int_0^1 x\ e^{-x^2}$ [5]

   (e) $\int (x+2)\sin(3x)$ dx [5]

   (f) $\int e^{\sqrt{x}}$ dx [5]

   (g) $\int \dfrac{x}{\sqrt{4-x^2}}$ [5]

10. (a) Sketch the graph of each function and shade the region whose area is represented by the integral below. **Label all pertinent information.** [5]

$$\int_{-3}^{4} (2x + 15) - x^2 \text{ dx}$$

Do not evaluate.

(b) Consider the region bounded by [5]

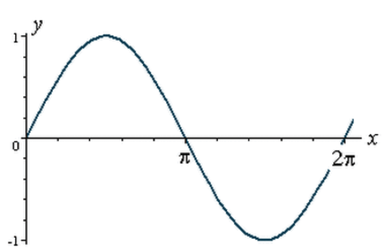
$$y = x^2, \ y = 2 - x^2.$$

1. Find the intersection points of the two curves.

2. Sketch the two curves and shade the region bounded by the two curves.

3. Set up, but **do not evaluate** an integral for the area of the shaded region.

11. Evaluate the area of the region bounded by the curves $y = \sin(x)$, $y = 0$, $x = 0$ and $x = \dfrac{\pi}{2}$. [5]
For your convenience, the graph of $y = \sin(x)$ is shown below.



12. Consider the following integrals and decide whether the best method of integration is integration by parts, u-substitution, or trig substitution. Explain the first key step/s of evaluating the integrals (There often are more than one right answer). **Do not evaluate the integrals.**

(a) $\int_0^4 \frac{\ln(x)}{\sqrt{x}}$ dx ________________________ [2]

(b) $\int \frac{1}{x\ln(x)}$ dx ________________________ [2]

(c) $\int_1^2 \ln(x)$ dx ________________________ [2]

(d) $\int xe^{0.2x}$ dx ________________________ [2]

(e) $\int_0^1 e^x \sin(x)$ dx ________________________ [2]

(f) $\int \frac{1}{x^2+2x+4}$ dx ________________________ [2]

13. Do the step-by-step work to show that

$$\int \frac{1}{x^2+25}\,\mathrm{dx} = \frac{1}{5}\arctan\left(\frac{x}{5}\right) + c.$$