This is a closed-book, closed-notes, no-calculators test. There are 61 points possible, but your score will be taken out of 60.

Fractions and roots in answers are fine; so are negative and fractional exponents.

Use scratch paper as needed, but any work that you want graded should be written legibly on this test paper.

Name:

(1 pt) Sign below to indicate your pledge. If your signature is difficult to read, please print your name as well.

I pledge that I will not give, accept, or tolerate others' use of unauthorized aid in completing this work.

_____

problems	max pts	total pts
problem 1& 2	9	
problems 3 & 4	9	
problems 5 & 6	9	
problem 7& 8	16	
problems 9	8	
problems 10,11,12, & 13	9	
Honor code pledge	1	
Total	61	

$y = f(x)$
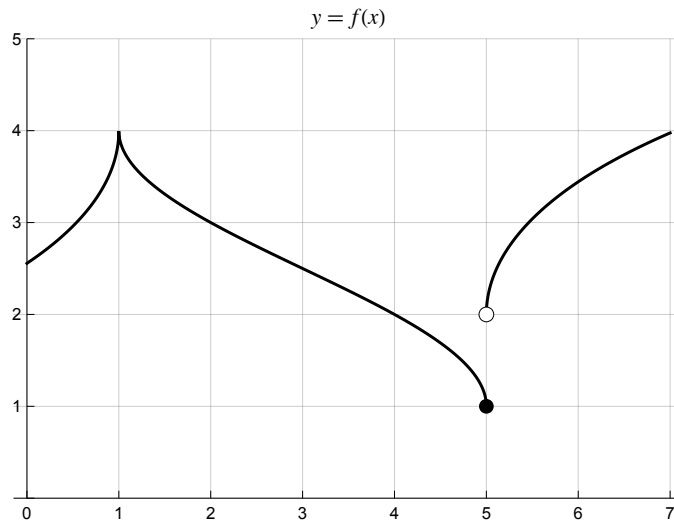
(3 pt) 1. Read the values of the following limits from the graph. Answer with $+\infty$ or $-\infty$ if appropriate.

a. $\displaystyle\lim_{x\to 5^-} f(x)$

b. $\displaystyle\lim_{x\to 1} f(x)$

c. $\displaystyle\lim_{x\to 3^+} \frac{f(x)}{3-x}$

(6 pt) 2. Use the graph of $f(x)$ shown above to answer the following:

a. What is the net change in f over the interval $[1,5]$?

b. What is the average rate of change in f over the interval $[1,5]$?

c. Which of the following is closest to the value of $f'(3)$? (Circle one value)

-3.1   -0.5   0.0   0.7   0.2

2

(6 pt) 3. Evaluate the following limits. Answer with $+\infty$ or $-\infty$ if it is appropriate.

a. $\displaystyle\lim_{t\to 0} \frac{t^2 - 4t}{t^2 + 8t}$

b. $\displaystyle\lim_{x\to 4} \frac{x^2 - 16}{x + 4}$

(3 pt) 4. Evaluate **ONE** of the following limits.
Cross out the one that you are not evaluating.

a. $\displaystyle\lim_{x\to 1} \frac{6x^3 + x^2 + 5x - 12}{x - 1}$ or   b. $\displaystyle\lim_{x\to 18} \frac{\sqrt{2x} - 6}{x - 18}$

(3 pt) 5. Give an example of a rational function with vertical asymptotes at both $x = 1$ and $x = 3$. (Write down a formula for the function.)

(6 pt) 6. True/False. Partial credit: 5 correct = 4 points; 4 correct = 2 points.

You are given that $f(x)$ is a polynomial of degree 4, $f(0) = -10$, and $f(80) = 30$.

_____ a. $f(x)$ must have the same average rate of change over every interval $[a, b]$.

_____ b. There must be at least one value c in $(0, 80)$ where $f'(c) > 0$.

_____ c. There must be at least one value c in $(0, 80)$ where $f'(c) = 0$.

_____ d. $f(x)$ must have at least one real zero.

_____ e. $f'(x)$ must have at least one real zero.

_____ f. f must be differentiable everywhere.

(8 pt) 7. Find an equation for the tangent line to $y = x^3 - x$ at $x = 2$.

(8 pt) 8. The position function for a departing train as it leaves the station is given by

$$f(t) = \frac{5}{2}t^2 + 4t$$

meters, at time t seconds. Determine the time at which the train reaches a velocity of 54 m/s, and determine the train's position at that time.

(8 pt) 9. The graph of $f(x) = x^3 + x^2 - x - 3$ is shown here:

$$y = x^3 + x^2 - x - 3$$
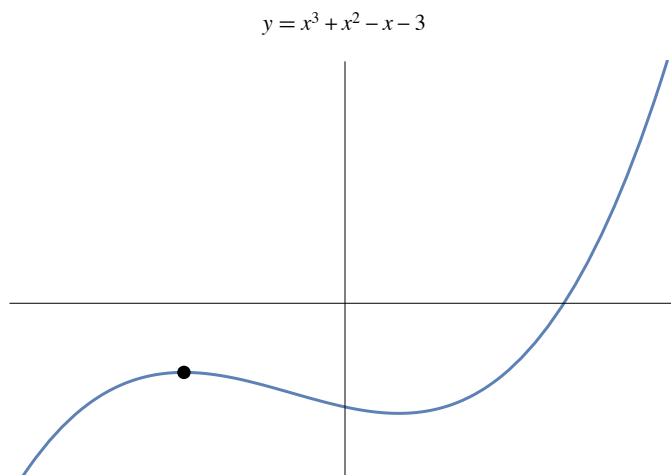
a. Compute $f''(x)$.

b. This function has two turning points. Find the exact x- and y-coordinates of the leftmost turning point (the one which is marked on the graph).

(3 pt) 10. Compute $f'(2)$ if $f(x) = \dfrac{1}{x^3}$

(2 pt) 11. Suppose f is a differentiable function, and $g(x) = x^2 f(x)$.
Express $g'(x)$ in terms of f and/or f' (just circle the letter of your choice):

a. $x^2 f'(x) + 2x f(x)$ b. $f'(x^2) f'(x)$ c. $2x f'(x)$ d. $2x f'(1)$ e. $2x + f'(x)$

(2 pt) 12. Suppose f is a differentiable function. Then $\dfrac{d}{dx}\left[f\left(x^2\right)\right] = \cdots$

a. $f'(x^2)$ b. $2x f'(x^2)$ c. $2x f'(x)$ d. $f(2x)$ e. $f'(2x)$

(2 pt) 13. Suppose y is a function of x (as we would do in implicit differentiation).

Choose the correct expression for $\dfrac{d}{dx}\left[x^3 y^3\right]$ from the following:

a. $3x^2 y^3$

b. $3x^2 y^2 \dfrac{dy}{dx}$

c. $x^3(3y^2) + y^3(3x^2)$

d. $x^3(3y^2)\dfrac{dy}{dx} + y^3(3x^2)$

e. $3x^2 + 3y^2 \dfrac{dy}{dx}$