

Last updated on 2016/10/17 at 10:45:57. Please inform me of mistakes.

Do not do the exercises in order. I suggest that you first complete a couple exercises from each topic. If you have more time, do more exercises from each topic.

0.1 Topic: Evaluating limits

1. Let $F(t)$ be a function defined by

$$F(t) = \frac{t^2 + 3t - 1}{5}.$$

- (a) Evaluate $\lim_{t \rightarrow 0} F(t)$
- (b) Evaluate $\lim_{t \rightarrow 1} F(t)$
- (c) Identify any points at which F is discontinuous.
2. Evaluate $\lim_{t \rightarrow 0} 7 + x - \sqrt{7} + \frac{x + 2}{x + 1}$.
3. Evaluate $\lim_{t \rightarrow 0^+} \frac{\sqrt{7 + x} - \sqrt{7 - x}}{2x}$.
4. Evaluate $\lim_{t \rightarrow 1^-} \frac{\sqrt{2x + 1} - \sqrt{3}}{x - 1}$.

0.2 Topic: Identifying Vertical asymptotes analytically

Identify the vertical asymptotes (if any) of following functions. If the function has no vertical asymptotes, say so.

- 1.

$$f(t) = \frac{5}{t^{10} - 2t^9}.$$

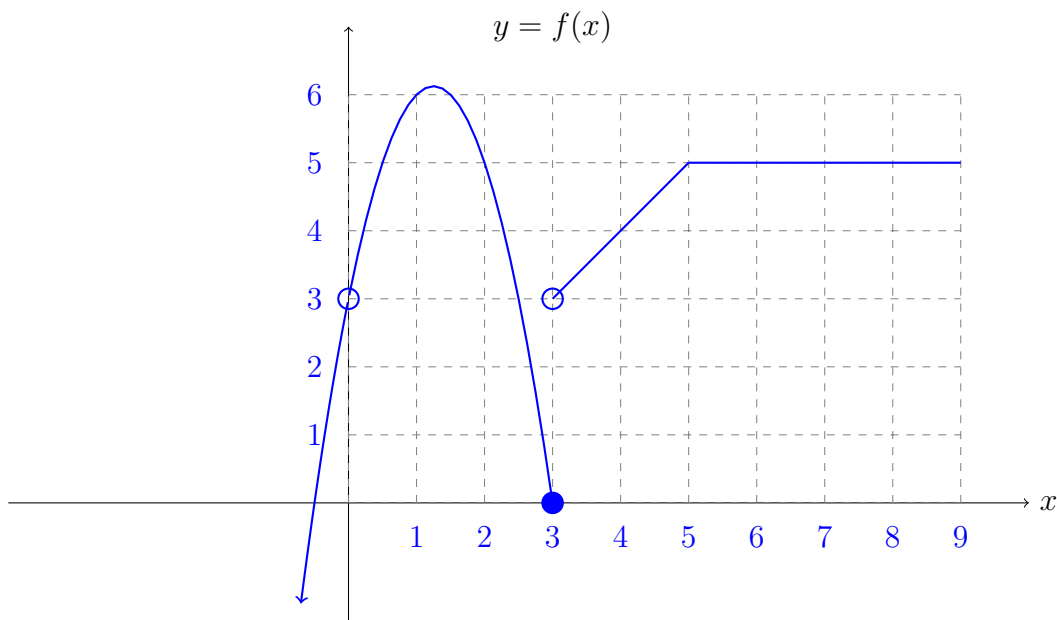
- 2.

$$g(x) = \frac{x + 1}{(x - 5)(x^3 + 1)}.$$

3. $h(x) = x^2 - 25x^2 + x + 3$.

0.3 Topic: Reading and sketching graphs

The figure shows the graph of a function $f(x)$.



Read the value graph; if it does not exist or is not defined, just say so.

1. $\lim_{x \rightarrow 0^-} f(x)$
2. $\lim_{x \rightarrow 0^+} f(x)$
3. $\lim_{x \rightarrow 0} f(x)$
4. $\lim_{x \rightarrow 3^-} f(x)$
5. $\lim_{x \rightarrow 3^+} f(x)$
6. $\lim_{x \rightarrow 3} f(x)$
7. $\lim_{x \rightarrow 4^-} f(x)$
8. $\lim_{x \rightarrow 4^+} f(x)$
9. $\lim_{x \rightarrow 4} \frac{f(x)}{11}$
10. $\lim_{x \rightarrow 5^-} f(x)$
11. $\lim_{x \rightarrow 5^+} f(x)$
12. $\lim_{x \rightarrow 5} \frac{f(x)}{x}$
13. $\lim_{x \rightarrow 7^-} f(x)$
14. $\lim_{x \rightarrow 7^+} f(x)$
15. $\lim_{x \rightarrow 7} \frac{x+1}{f(x)}$

16. $f(0)$
17. $f(3)$
18. $f(4)$
19. $f(5)$
20. $f(6)$
21. $f(7)$
22. For each part, answer True / False
 - (a) f is continuous on $[0,2]$.
 - (b) f is continuous on $(0,2)$.
 - (c) f is continuous on $[1,3]$.
 - (d) f is continuous on $(1,3)$.
 - (e) f is continuous on $[3,5]$.
 - (f) f is continuous on $(3,5)$.
 - (g) f is continuous on $[3,6]$.
 - (h) f is continuous on $(3,6)$.
 - (i) f is continuous on $[4,6]$.
 - (j) f is continuous on $(4,6)$.
23. For each part, answer True / False
 - (a) $\lim_{x \rightarrow 0^-} f(x) = f(0)$.
 - (b) f is continuous at 0.
 - (c) f has a removable discontinuity at 0.
 - (d) f has a non-removable discontinuity at 0.
 - (e) $\lim_{x \rightarrow 2^+} f(x) = f(2)$.
 - (f) f is continuous at 2.
 - (g) f has a removable discontinuity at 2.
 - (h) f has a non-removable discontinuity at 2.
 - (i) $\lim_{x \rightarrow 3^-} f(x) = f(3)$.
 - (j) f is continuous at 3.
 - (k) f has a removable discontinuity at 3.
 - (l) f has a non-removable discontinuity at 3.
 - (m) $\lim_{x \rightarrow 5} f(x) = f(5)$.
 - (n) f is continuous at 5.
 - (o) f has a removable discontinuity at 5.
 - (p) f has a non-removable discontinuity at 5.

- (q) f is continuous on $(1,3)$.
- (r) f is continuous on $[3,5]$.
- (s) f is continuous on $(3,5)$.
- (t) f is continuous on $[3,6]$.
- (u) f is continuous on $(3,6)$.
- (v) f is continuous on $[4,6]$.
- (w) f is continuous on $(4,6)$.
24. For each part, set up suitable axes and sketch the graph of a function satisfying all the given conditions. Make your drawing as clear as possible. If I cannot tell whether it's correct, I'll assume it's not.
- (a) $f(1)=2$; f has a removable discontinuity at 1, but f is continuous everywhere else.
- (b) $f(1)$ is not defined; f has a removable discontinuity at 1, but f is continuous everywhere else.
- (c) $f(1)=2$; f has a non-removable discontinuity at 1, but f is continuous everywhere else.
- (d) $f(1)$ is not defined; f has a non-removable discontinuity at 1, but f is continuous everywhere else.
- (e) f is continuous everywhere except at $x = 2$ and $x = 5$, and $\lim_{x \rightarrow 2} f(x) = +\infty$ and $\lim_{x \rightarrow 5} f(x) = -\infty$.
- (f) f is continuous on $(2, 5)$, and $\lim_{x \rightarrow 2^+} f(x) = +\infty$ and $\lim_{x \rightarrow 5^-} f(x) = +\infty$.
- (g) f is continuous on $(2, 5)$, and $\lim_{x \rightarrow 2^+} f(x) = +\infty$ and $\lim_{x \rightarrow 5^-} f(x) = -\infty$.
- (h) f is continuous on $(2, 5)$, and $\lim_{x \rightarrow 2^+} f(x) = -\infty$ and $\lim_{x \rightarrow 5^-} f(x) = -\infty$.

0.4 Topic: Intermediate Value Theorem

- Fill each blank with either an English word or a mathematical symbol so that the following is a complete and accurate statement of the Intermediate Value Theorem.

If a _____ $f(x)$ is _____ on the _____ interval $[a, b]$, and a number M is between ____ and ____, then there is at least one point c in _____ where _____ = ____.

- For each part, answer either True or False. For all parts, assume that f is continuous on $[0, 2]$, that $f(0) = -10$ and $f(2) = 10$.
 - There must be at least one value c in $(0, 2)$ where $f(c) = 10$.
 - There must be at least one value c in $[0, 2]$ where $f(c) = 10$.
 - There must be at least one value c in $[0, 2]$ where $f(c) = -1$.
 - There must be at least one value c in $[0, 2]$ where $f(c) = -20$.
 - $f(1)$ must be between -10 and 10 .

0.5 Topic: Evaluating limits (part 2)

Evaluate the following limits. Whenever appropriate, answer with $+\infty$ or $-\infty$.

1. $\lim_{x \rightarrow 2^+} \frac{x}{x-2}$.

2. $\lim_{x \rightarrow 2} \frac{1-x}{(x-2)^2}$.

3. $\lim_{x \rightarrow 5^-} \frac{5-x}{(x-2)^2}$.

4. $\lim_{x \rightarrow 5^+} \frac{5+x}{(x-2)^2}$.

5. $\lim_{x \rightarrow -1^+} \frac{x^3+1}{x+1}$.