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1. Let $F(t)$ be a function defined by $F(t) = t^2 + 3t - 1$.

   (a) Evaluate the *difference quotient*

   $$\frac{F(t + \Delta t) - F(t)}{\Delta t}.$$

   (b) Compute the *net change* in F over the interval $[-1, 4]$.

   (c) Compute the *average rate of change* in F over the interval $[-1, 4]$.

   (d) Write an equation for the secant line which meets the graph of f at $x = -1$ and $x = 4$.

   (e) Write an equation for the secant line which meets the graph of f at $x = a$ and $x = 4$, where a is a real number that is smaller than 4.

   (f) Write an equation for the secant line which meets the graph of f at $x = a$ and $x = b$, where a and b are real numbers such that $a < b$.

2. Function g is defined by

   $$g(x) = \frac{(x - 4)^{999} + 2x}{70}.$$

   (a) Evaluate the following and simplify as much as possible. Simplified fractions are fine; don't get decimals: $g(5) - g(4)$

   (b) What is the *average rate of change in* g over the interval $[4, 5]$?

   (c) Write an equation for the secant line which meets the graph of g at $x = 4$ and $x = 5$.

   (d) Evaluate the following and simplify as much as possible. Simplified fractions are fine; don't get decimals: $g(x + 4)$

3. (a) Give an example of a polynomial of degree 7 which falls to the left, rises to the right, and has x=2, x=5, and x=11 as zeros. You do not need to sketch the graph, but you need to write the formula for the polynomial.

   (b) Give an example of a polynomial of degree 5 which rises to the left, falls to the right, and has x=2 as a zero. You do not need to sketch the graph, but you need to write the formula for the polynomial.

   (c) Give an example of a polynomial of degree 6 which falls to the left, falls to the right, and has x=2 as a zero. You do not need to sketch the graph, but you need to write the formula for the polynomial.

   (d) True or False: every polynomial of even degree (2 or higher) has at least a zero. (If you write True, give an explanation. If write False, give a counterexample. If you give a counterexample, you need to give the formula for the polynomial.)

   (e) True or False: every polynomial of odd degree has at least one zero. (If you write True, give an explanation. If write False, give a counterexample. If you give a counterexample, you need to write the formula for the polynomial.)

   (f) True or False: If the average rate of change in a function f over $[a, b]$ is positive, then f must be increasing over $[a, b]$.

   (g) True or False: If f is a linear function, then f has the same net change over every possible interval.

   (h) True or False: If f is a linear function, then f has the same average rate of change over every possible interval.

   (i) Which of the following functions has/ have a constant rate of change (the same average rate of change over every possible interval)?
   a. $\frac{1}{9}x$ b. $\frac{9}{x}$ c. $\frac{1}{9x}$ d. $(\frac{1}{9}x - 1)(x + 9)$ e. $9x - \frac{1}{9}$

4. Find the zeros of the following functions. If no zeros exist, say so.

   (a) $f(t) = t^{10} - 2t^9$.

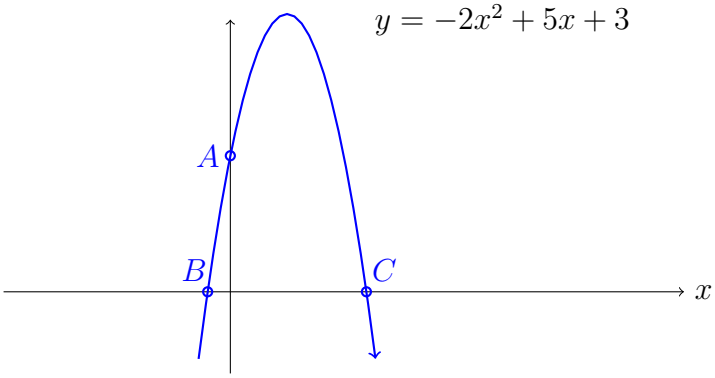
   (b) $h(x) = (x-2)(x^2+1)$. (Note: you saw this yesterday).

   (c) $g(x) = (x-5)(x^3+1)$. (Note: you saw this yesterday).

   (d) $j(x) = (x+5)^3(x^2-2)(x-\frac{1}{2})$.

   (e) $h(x) = x^2 + x + 3$.

5. The figure shows the graph of a quadratic function:



   (a) Find the *exact* coordinates of each of the three marked points A, B, and C. Note, A is the point where the graph crosses the y-axis; B and C are the points where it crosses the x-axis.

   (b) Write down exactly all the *zero/s* (aka *root/s*) of this function.

   (c) What degree is this polynomial? What is the leading term of this polynomial?

   (d) How many turning points does this graph have?

(e) Write an equation of the secant line between the points $(0, 3)$ and $(2, 5)$.

(f) Write an equation of the secant line between the points $(1, 6)$ and $(2, 5)$.

6. Let $f(x) = x^3 - 2$.

(a) Evaluate $f(x + \Delta x)$ (your answer should be a simple formula involving two variables x and Δx.(your answer should be a simple formula involving two variables x and Δx.

(b) Evaluate and simplify the following quotient as much as possible.

$$\frac{f(x + \Delta x) - f(x)}{\Delta x}.$$