Limit calculations can require a lot of steps involving simplification and changes of form before it's finally possible to *evaluate* the limit. That means it's important to write clearly, so that the reader can follow your steps. Not only that, writing clearly increases your chances of arriving at a correct answer, and increases the usefulness of your work – you'll be able to look back at it later, when you review, and understand what you did.

Here are three guidelines specifically about writing limits:

1. Don't drop the limit notation until you have actually evaluated the limit (typically by direct substitution) or determined that the limit does not exist. For example, this is incorrect:

$$\text{[WRONG]} \qquad \lim_{x \to 1} \frac{x^3 - 1}{x - 1} = \frac{(x-1)(x^2 + x + 1)}{(x - 1)} = \text{etc} \ldots$$

It should be written

$$\text{[Right]} \qquad \lim_{x \to 1} \frac{x^3 - 1}{x - 1} = \lim_{x \to 1} \frac{(x-1)(x^2 + x + 1)}{(x - 1)} = \lim_{x \to 1}(x^2 + x + 1) = 3$$

The first version is wrong because it's confusing the limit of a thing with the thing itself – they're not the same! Practically speaking, it leads to errors, because students who drop the limit sign early in a problem often forget, at the end, that they're supposed to do an evaluation step (like letting x go to 1), because they've stopped writing that part. And from a reader's perspective, it makes the chain of equations harder to follow ("Weren't we taking a limit? What happened to that?").

The flow of the correct version, when read aloud, is "This limit is equal to this limit, which is equal to this simpler limit, which we can finally evaluate by direct substitution: it's 3." It's a long sentence, but it is correct and meaningful, and not hard to follow.

2. Don't write "unattached" limit signs. For example,

[WRONG] $\qquad \lim_{x \to 1} = 4$

is an incorrect use of the notation, and unacceptable. You might write

[Right] $\qquad \lim_{x \to 1} f(x) = 4 \qquad$ or $\qquad \lim_{x \to 1} \frac{x+3}{x} = 4,$

but a limit is always a limit *of* something. In the [WRONG] example, I would ask, "The limit of *what*, as x approaches 1? What are you taking the limit of?" It's not clear.

This causes difficulties, because a multistep problem may have more than one limit calculation in it. If you need to look back to find the value of a limit from an earlier step, a statement like the [WRONG] example won't give you any clue what limit you were evaluating.

Every **lim** sign you use should have something beneath it (like $x \to 1$) to indicate what variable is approaching what point, and and a function immediately following it to indicate what you're taking the limit *of*.

3. Don't write ungrammatical things, even when you're using mathematical symbols. For example,

[WRONG] $\qquad \lim\limits_{x \to 2} \dfrac{1}{x-2} = \text{does not exist}$

would be read aloud as, "The limit of $\frac{1}{x-2}$, as x approaches 2, equals does not exist." That's both wrong (the limit doesn't *equal* anything), and not even valid English. If you mean

[Right] $\qquad \lim\limits_{x \to 2} \dfrac{1}{x-2}$ does not exist,

then say that! Or if you'd prefer to say

[Right] $\qquad \lim\limits_{x \to 2} \dfrac{1}{x-2}$ is undefined,

that's a fine way to say it too. Both of those can read aloud as proper, grammatical sentences. The first version cannot.

Two more comments, broadly speaking (not just about limits):

4. If you can't read what you've written aloud, in grammatical English, then (1) it's probably nonsense, and (2) you've written something that you yourself don't understand, which can't possibly be acceptable to either of us.

5. Don't put the burden on the reader. When you're writing math for me, don't assume, "The professor will figure out what I mean even if I can't express it clearly." Treat me as if I need things explained very clearly.

Remember, I already know the answers. I don't want to fish around in your work to see if you might have done something relevant to finding the answer. It's your responsibility to present your work clearly.

Your goal is to write a solution which is sufficiently clear and organized that someone who does not yet know how to solve the problem could learn to solve it by reading what you've written.