The following problems to be turned in. You may work alone or in a group of two; if you prefer to work in a pair, then **please work with a different person** than you did on Problems 1. Submit just one solution write-up with both names on it. You're welcome to ask for help or hints in office hours. If you ask the calculus tutors for help, they will expect to see your solutions to the skill exercises on the previous page first.

Write a final draft of your solutions on clean paper (without ragged edges), leave me a generous amount of space around each problem to write comments, and staple multiple pages. Due at the beginning of class Tuesday, October 4th.

(15 pt) 1. Linda jogs back and forth on a path in Central Park for 30 minutes. Suppose her distance s in feet from the oak tree on the north side of the park t minutes after she begins her jog is given by the function $s(t)$ whose graph is shown below. (Imagine that she jogs on a straight path leading into the park from the oak tree.)

a. What is the average rate of change of Linda's distance from the oak tree over the entire 30-minute job? What does this mean in real-world terms?

Answer:

$$\frac{s(30)-s(0)}{30-0} = \frac{0}{30} = 0.$$

This means that, during these 30 minutes, Linda's has been moving 0 foot per minute (on average).

b. On which 10-minute interval is the average rate of change of Linda's distance from the oak tree the greatest: the first 10 minutes, the second 10 minutes, or the last 10 minutes? (Explain!)

Answer: both the first and second 10 minutes. We can estimate the average rate of change for each of the three intervals, as written below.

interval	average rate of change (in feet/minute)
first 10 minutes	$\frac{s(10)-s(0)}{10-0}$ is roughly $\frac{150}{10}$
second 10 minutes	$\frac{s(20)-s(10)}{20-10}$ is roughly also $\frac{150}{10}$
last 10 minutes	$\frac{s(30)-s(20)}{30-20}$ is roughly $\frac{0-300}{10} = -30$

c. Use the graph of $s(t)$ to estimate Linda's average velocity during the 5-minute interval from $t = 5$ to $t = 10$. (Read the necessary values from the graph as accurately as you can, and use a calculator to get a decimal value for the average velocity. Include appropriate units with your answer).

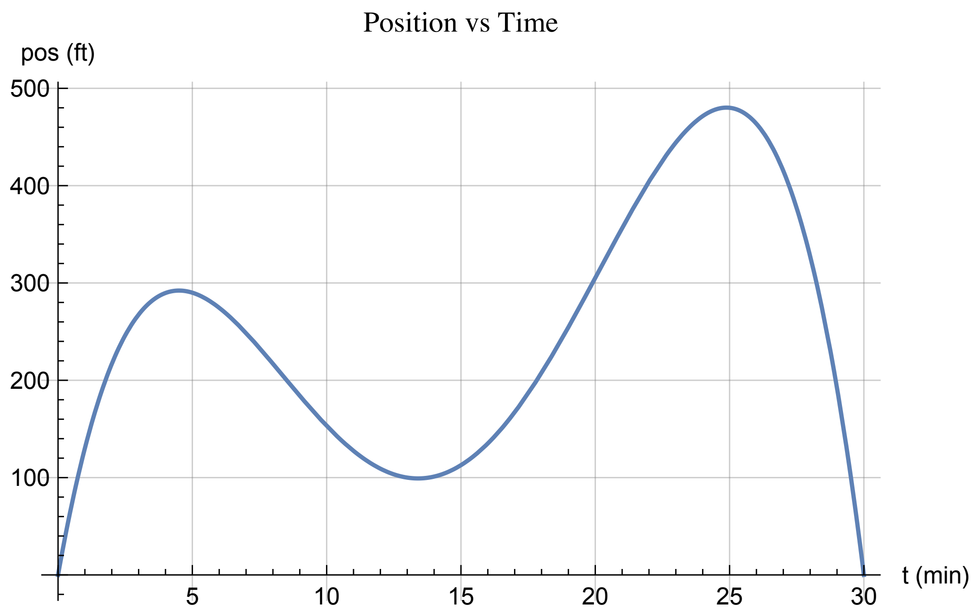
Answer: Roughly, $s(5) = 290$ and $s(10) = 150$, so Linda's average velocity during this 5-minute interval is roughly $\frac{150-290}{10-5}$ feet per minute, which is equal to -28 feet per minute.

d. Approximate the times t at which Linda's *velocity* is equal to zero. What is the physical significance of these times?

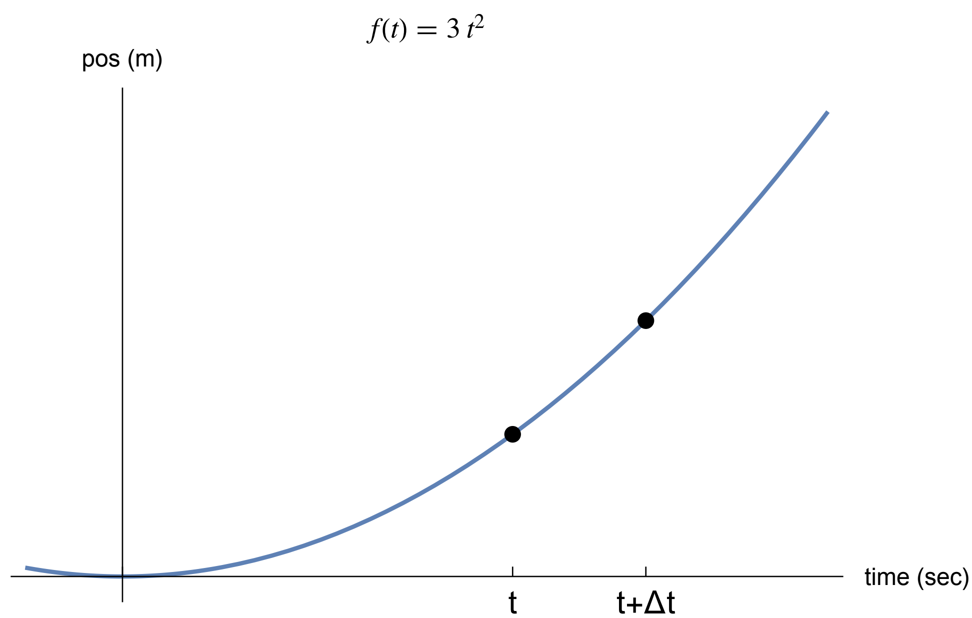
Answer: approximately near $t = 4$, $t = 13$, and $t = 25$. At each of these times, Linda is making a 180-degree turn. For example, from $t = 0$ to around $t = 4$, Linda is going away from the tree, but, at around $t = 4$, Linda starts going back toward the tree.

e. Give two examples of time intervals on which Linda's position function $s(t)$ is decreasing.

Answer: Some possible time intervals are $[5, 10], [5, 6], [27, 30]$, etc.



(15 pt) 2. The graph below is for a position-vs-time function $f(t)$ for a moving object. We also happen to have a formula for the position function: $f(t) = 3t^2$.



a. Find the slope of the secant line through the two marked points on the graph, in terms of t and Δt.
Your answer should be a formula involving the two variables t and Δt.
Explanation: Recall that (if $a < b$) the slope of the secant line through two points $(a, f(a))$ and $(b, f(b))$ is the same as the average rate of change in f over the interval $[a, b]$, which is

$$\frac{f(b) - f(a)}{b - a}.$$

We see that $t + \Delta t$ is larger than t (since we can see that $t + \Delta t$ is to the right of t), so in this situation it may help you to imagine $a = t$ and $b = t + \ \Delta t$.
Answer: The slope is (before any simplification)

$$\frac{f(t+\ \Delta t)-f(t)}{(t+\ \Delta t)-t}\ .$$

b. Simplify the slope from part (a) as much as possible: expand a power, combine like terms, and cancel a common factor from the fraction. Show your steps neatly (so that a not-too-skilled reader could follow what's going on at each step.)

Your answer should still be formula involving the two variables t and Δt.

Simplification steps:

$$\frac{f(t+\ \Delta t)-f(t)}{(t+\ \Delta t)-t}=\frac{3\ (t+\ \Delta t)^2-3\,t^2}{\Delta t}=3\ \frac{(t+\ \Delta t)^2-\ t^2}{\Delta t}=3\ \frac{(t^2+\ 2t\ \Delta t\ +(\Delta t)^2)-\ t^2}{\Delta t}$$

$$=3\ \frac{2t\ \Delta t\ +(\Delta t)^2}{\Delta t}=3\ \Delta t\ \frac{2t+(\Delta t)}{\Delta t}=3(2t+\Delta t)=6t+3\ \Delta t\,.$$

c. Give an *interpretation* of your formula from part (b): In terms of physical motion, what does it represent? What are the appropriate units for it?

Answer: the formula from part (b) represents the average velocity of the object time t and the next Δt seconds. The appropriate unit would be meters per second.

d. Evaluate the limit of your answer from part (b) as the variable Δt approaches zero. (Your answer should be a formula involving only the variable t.)

Explanation:

we can think of the formula from part (b) as a new function,

$$g(\Delta t)=6t+3\ \Delta t$$

where Δt takes the usual role of the "input" "x", and think of t as a fixed unknown constant (not an input variable!)

This new function is a polynomial function of the variable Δt, so it is continuous everywhere (including at $\Delta t=0$). Therefore, the limit of $g(\Delta t)$ as Δt approaches 0 is equal to the value $g(0)$.

We can compute the value of $g(0)$ by just replacing the variable Δt with 0, so we compute

$$g(0)=6t+3(0)=6t.$$

Answer: Therefore, the limit of $g(\Delta t)$ as Δt approaches 0 is $6t$.

e. Give an *interpretation* of your formula from part (d). Use your own reasoning (and your own words) to explain what this formula represents in terms of physical motion.

Answer: The formula $6t$ represents the speed of the moving object at time t. For example, at time 0, the speed is 0 meter per second. Then, 3 seconds later, the object is moving at the speed of 18 meters per second.