## Math 118 Calculus Ia

## Problems 2

The skill practice questions on this page are *not* to be turned in. Just work them for practice before you begin the problems on the other side.

0.1 Evaluate the following expressions, using the functions defined by  $f(x) = x^3 - 4$  and  $g(t) = 3(1-t)^2$ .

- a. f(x+4)
- b.  $g(t + \Delta t)$
- c.  $f(x \Delta x)$
- d.  $g(r^2 + r + 1)$

0.2 Find the slope of the secant line meeting the graph of the function f at the given x values:

a.  $f(x) = x^3 - 7x$  at x = -1 and x = +2

- b.  $f(x) = x^2 x$  at x = 0 and x = r (your answer will be in terms of r).
- c.  $f(x) = (x+1)^2$  at x = a and x = a+1 (your answer will be in terms of a).

0.3 Fully expand each power, and combine like terms in the expansion.

- a.  $(x+1)^3$
- b.  $(a+2b)^2$
- c.  $(q + \Delta q)^3$

Don't panic; q and  $\Delta q$  are just two variable names, just like a and b in the previous part.

## 0.4. Simplify (by cancelling a common factor).

If the numerator and denominator have no factor in common, leave the expression as it is.

a. 
$$\frac{x^{2} + x - 2}{x^{2} + 2x - 3}$$
  
b. 
$$\frac{6x^{2}\Delta x + 12x\Delta x^{2} + 8\Delta x^{3}}{3\Delta x}$$
  
c. 
$$\frac{t^{3} + t^{2} + 4}{t^{2} + t + 2}$$
  
d. 
$$\frac{2\left(3h + h^{3} - 6hq + 3hq^{2}\right)}{3h^{2}}$$

The following problems to be turned in. You may work alone or in a group of two; if you prefer to work in a pair, then **please work with a different person** than you did on Problems 1. Submit just one solution write-up with both names on it. You're welcome to ask for help or hints in office hours. If you ask the calculus tutors for help, they will expect to see your solutions to the skill exercises on the previous page first.

Write a final draft of your solutions on clean paper (without ragged edges), leave me a generous amount of space around each problem to write comments, and staple multiple pages. Due at the beginning of class Tuesday, October 4th.

(15 pt) 1. Every morning Linda jogs back and forth on a path in Central Park for 30 minutes. Suppose her distance s in feet from the oak tree on the north side of the park t minutes after she begins her jog is given by the function s(t) whose graph is shown below. (Imagine that she jogs on a straight path leading into the park from the oak tree.)

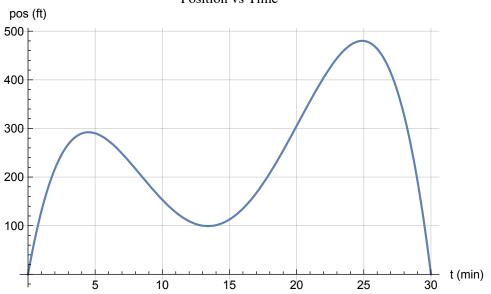
a. What is the average rate of change of Linda's distance from the oak tree over the entire 30-minute job? What does this mean in real-world terms?

b. On which 10-minute interval is the average rate of change of Linda's distance from the oak tree the greatest: the first 10 minutes, the second 10 minutes, or the last 10 minutes? (Explain!)

c. Use the graph of s(t) to estimate Linda's average velocity during the 5-minute interval from t = 5 to t = 10. (Read the necessary values from the graph as accurately as you can, and use a calculator to get a decimal value for the average velocity. Include appropriate units with your answer).

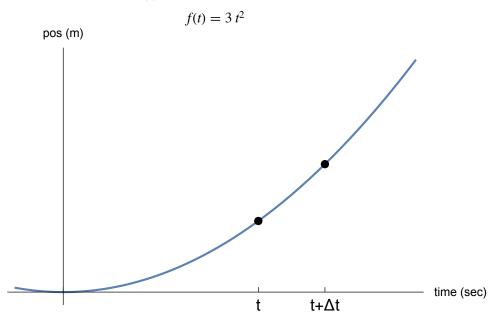
d. Approximate the times t at which Linda's *velocity* is equal to zero. What is the physical significance of these times?

e. Give two examples of time intervals on which Linda's position function s(t) is decreasing.



## Position vs Time

(15 pt) 2. The graph below is for a position-vs-time function f(t) for a moving object. We also happen to have a formula for the position function:  $f(t) = 3t^2$ .



a. Find the slope of the secant line through the two marked points on the graph, in terms of t and  $\Delta t$ . Your answer should be a formula involving the two variables t and  $\Delta t$ .

b. Simplify the slope from part (a) as much as possible: expand a power, combine like terms, and cancel a common factor from the fraction. Show your steps neatly (so that a not-too-skilled reader could follow what's going on at each step.)

Your answer should still be formula involving the two variables t and  $\Delta t$ .

c. Give an *interpretation* of your formula from part (b): In terms of physical motion, what does it represent? What are the appropriate units for it?

d. Evaluate the limit of your answer from part (b) as the variable  $\Delta t$  approaches zero. (Your answer should be a formula involving only the variable t.)

e. Give an *interpretation* of your formula from part (d). Use your own reasoning (and your own words) to explain what this formula represents in terms of physical motion.