1. Find the maximum and minimum values of $f(x) = 2x^3 - 15x^2 + 24x$ on the interval $[2, 5]$.

Identify *where* the max and min occur (that is, at what x-values), and what the max and min values are.

(One team member may take out their calculator to compute large values)

Answer: By the Extreme Value Theorem, the max and min do occur in the closed interval $[2, 5]$. We know that an extreme value has to occur on either a critical point or on an endpoint.

First, we compute $f'(x) = 6x^2 - 30x + 24 = 6(x^2 - 5x + 4) = 6(x - 1)(x - 4)$.

Next, we set $0 = f'(x)$ and solve. We get that either $x = 1$ or $x = 4$. Note that 1 is not in our interval $[2, 5]$, but $x = 4$ is.

We compute $f(2) = 4$, $f(4) = -16$, $f(5) = -5$. So the minimum value is -16 and it occurs at $x = 4$. The maximum value is 4 and it occurs at $x = 2$.

2. A box is to be made from a flat, 8.5 inches x 11 inches sheet of metal by cutting equal squares from the corners, folding up the sides, and welding them together. The resulting box has a bottom and four sides, but no top.

a. You and your team members are to make three boxes out of paper such that the height of the box is:

i. 1 inches; ii. 2 inches; iii. 3.5 inches.

b. Approximate the order of the volumes of the three boxes (from smallest to largest) by filling the boxes with balls. Do not use arithmetic yet.

c. Use arithmetic to compute (precisely) the order of the volumes of the three boxes (from smallest to largest). (One team member may take out their calculator to do multiplication on 2-digit integers)

Answer: I use the answer from part e to compute $V(1) = 58.5$, $V(2) = 63$, and $V(2.5) = 52.5$.

d. Make a sketch of the sheet of metal with the corners cut out. Choose a variable name to represent the size of the length of the square that has been cut out, and put it into your sketch in the appropriate places.

Answer: I choose x to represent the length of the missing square.

e. Find the maximum possible volume for the box. Indicate clearly what the volume of the optimal box will be, what its dimensions will be (length, width, and height), and what size square we should cut from the corners in order to construct it.

Answer: First, figure out the appropriate function for the volume of the box, which is:

$V(x) = x(11 - 2x)(8.5 - 2x) = 4x^3 - 39x^2 + 93.5x$ cubic inches.

The possible values for x (so that we can make a box) is between 0 inches and 4.25 inches.

To maximize V, we find the critical numbers of this volume function:

We compute $V'(x) = 12x^2 - 78x + 93.5$.

We solve $V'(x) = 0$, and get

$x = (78 + \sqrt{78^2 - 4(12)(93.5)})/24 = (78 + \sqrt{1596})/24$, which is about 4.91

or $x = (78 - \sqrt{78^2 - 4(12)(93.5)})/24$, which is about 1.58.

Note that only one of these numbers ($x = 1.58$) is in our feasible domain $[0, 4.25]$, so we only need to check the volume when $x = 1.58$ and also on the endpoints.

We check that $V(0) = 0$ and $V(4.25) = 0$ and $V(1.58)$ is approximately 66.147.

3. Let $f(x) = (x^2 - 5x)(x - 10)$.

a. Find $f'(x)$.

b. Find the value of x which maximizes $f(x)$ on the interval $[0, 5]$.

Graph this using a computing device (desmos, wolframalpha, your graphing calculator), and use a computing device to find the critical numbers of f.

The critical numbers of f are approximately $x = 2.1$ and $x = 7.89$. You don't need to consider $x = 7.89$ because this is outside our domain.

We find that the minimum value of f is equal to 0, and it happens at $x = 0$ and $x = 5$. We find that the maximum value of f is equal to approximately 48.1125, and it happens at $x = 2.1$.

4. Let $f(x) = 2x^3 - 9x^2 + 12x - 100$.

a. Find $f'(x)$.

Answer: $f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2) = 6(x - 1)(x - 2)$.

b. Find the value of x which maximizes $f(x)$ on the interval $[0, 3/2]$.

Answer: The critical numbers of f are $x = 1$ and $x = 2$. We don't need to consider $x = 2$ because that is outside our interval $[0, 3/2]$.

We check that $f(0) = -100$, $f(3/2) = -95.5$, and $f(1) = -95$. So the value of x which maximizes $f(x)$ on the interval $[0, 3/2]$ is $x = 1$.