The skill exercises on this page are not to be turned in, but you'll need these skills in order to succeed with the problems that follow. Take some time and make sure you can do these confidently before you go on.

0.1 Write the equation of a line that passes through (1,3), perpendicular to the given line.
Refer to p54 in the text for some help.

a. $y = 3(x-5)+1$

b. $2(y-1) = 7(x-5)$

c. $4y+7 = 4x+14$

0.2 Evaluate $f(a)$ and simplify the result as much as possible.
(You should be able to get an expression with at most one radical in it if you simplify thoroughly.)

a. $f(x) = x^2 - 2x - 1$, and $a = \left(2+\sqrt{5}\right)$

b. $f(x) = 8x^3 + 4x^2 - 4x - 1$, and $a = \sqrt{11}$

c. $f(x) = x^3 - 3x$, and $a = \dfrac{3}{\sqrt{7}}$

0.3 Find all solutions to the equation:

a. $\dfrac{5a+4}{6a+4} = \dfrac{-3}{5}$ b. $\dfrac{1+b^2}{5} = b$ c. $\dfrac{c^2+8}{7c+8} = \dfrac{3}{5}$

0.4 Write the equation of the tangent line to the graph of f at the given point:

a. $f(x) = x^3 + 11x$ at $x = 1$

b. $f(x) = x^3 + 11x$ at $x = a$ (Treat a as a constant.)

c. $f(x) = x^2 - 2x - 11$ at $x = -3$

d. $f(x) = x^2 - 2x - 11$ at $x = c$ (Treat c as a constant.)

0.5 Find the x-intercept of the line (that is, the point where it crosses the x-axis).

a. $y = 7(x-3)+9$

b. $3y+4 = 4x+19$

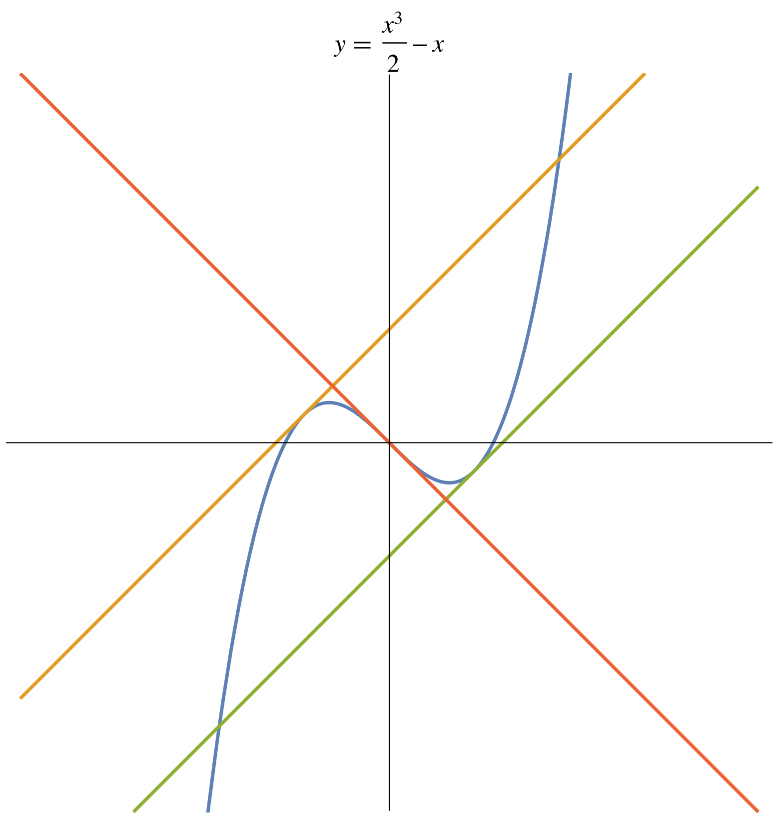
c. $y = k(x-3)+9$ (Treat k as a constant.)

d. $y = 2m(x-m)+m^2+4$ (And yes, treat m as a constant.)

The following problems are to be turned in. You may work alone or in a pair. If you **work in a pair** (which is **a good idea**), submit just one solution with both names on it. You're welcome to ask for help or hints in office hours. If you ask the calculus tutors for help, they will expect to see your solutions to the skill problems on the previous page first.

Write a final draft of your solutions on clean paper (without ragged edges), leave me a generous amount of space around each problem to write comments, and staple multiple pages. Due at the beginning of class Monday, November 21.

(10 pt) 1. The figure below shows the graph of the function $f(x) = (1/2)x^3 - x$ together with three of its tangent lines. Your goal will be to recreate recreate the figure, and find the exact location of some of the important points.



a. Write the equation of the (red) tangent line to the graph at the origin.

b. Read p54, *Parallel and Perpendicular Lines*

c. Given that the other two (green and orange) tangent lines are perpendicular to the first one, solve to find the exact location of the two points of tangency. Write the equations for these two tangent lines as well.

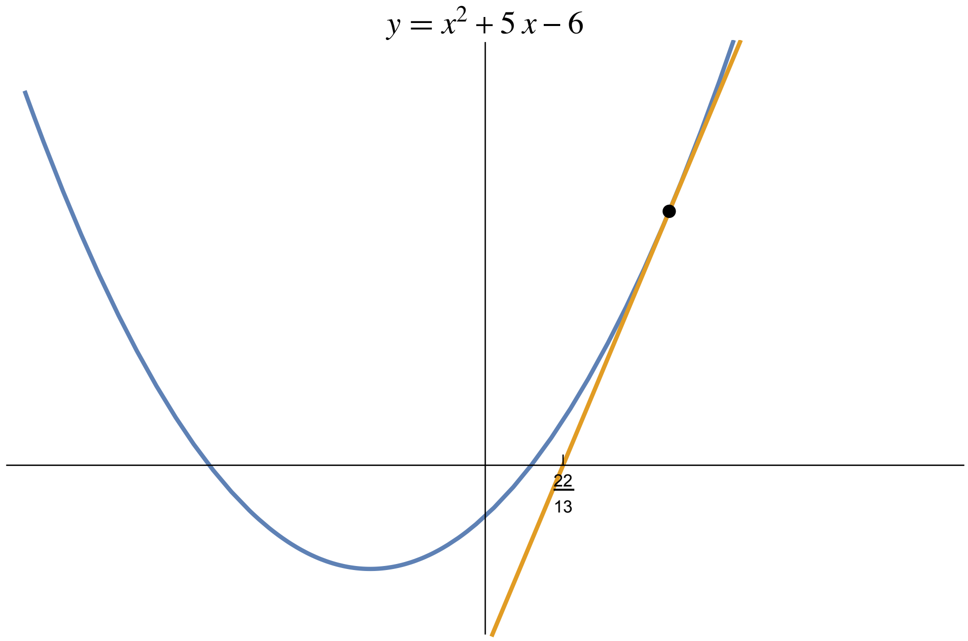
d. Using Desmos (or something similar), graph the function $f(x)$ together with all three tangent lines. Use a sensible range of $x-$ and y-values so that we see all the important points in the figure, and the graph fills most of the plot range without a lot of empty space.

Tip: Use the same range for the x and y values, and make sure that the image is close to square, so that the lines which are supposed to be perpendicular really look perpendicular.

Turn in a printout of the graph with your solution.

1e. (Optional, if you like a challenge). The green and orange tangent lines each intersect the graph of the cubic function in one additional point (besides the point of tangency). Solve to find the exact coordinates of these two intersection points, and add them to your graph.

(10 pt) 2. This figure shows the graph of $f(x) = x^2 + 5x - 6$ together with one of its tangent lines:



The point of tangency on the graph is unknown, but you know that the tangent line crosses the x-axis at exactly $x = 22/13$.

a. Set up and solve an equation that will help you find the point of tangency.

Both parts of the – "set up" and "solve" – will take some work. Your write-up should have a clean, easy-to-follow solution that shows your steps, and includes enough words of explanation that a reader can follow the process.

b. Write an equation for the tangent line at that point.

c. The parabola has another tangent line that crosses the x-axis at the same point. Find the other tangent line and write an equation for it as well.

d. Using Desmos (or other graphing technology), construct a plot showing the function $f(x)$, the two tangent lines, the point of tangency, and the common point where both tangent lines cross the x-axis.