The skill practice questions on this page are not to be turned in.

0.1. Simplify each of the following to an expression which has at most one radical in it (or no radical at all, if that's possible).

a. $5\sqrt{10} - 2\sqrt{10}$

b. $\left(\sqrt{3}\right)^4$

c. $\left(\sqrt[3]{11}\right)^4 + 2\sqrt[3]{11}$

d. $\left(\sqrt{13}\right)^5 - \left(\sqrt{13}\right)^3$

e. $\left(\sqrt{2}+\sqrt{3}\right)^4 - 10\left(\sqrt{2}+\sqrt{3}\right)^2 + 1$

0.2. Find the slope of each line:

a. The line through $(2, 5)$ and $(34, 55)$

b. The line $2y + x + 6 = 5y - 4x - 7$

c. The secant line cutting the graph of $f(x) = \dfrac{1}{1+x}$ at $x = 2$ and $x = 5$.

0.3. Find the derivative of the polynomial:

a. $f(x) = x^4 - 10x^2 + 1$

b. $g(t) = t^3 - 6t^2 + 12t - 11$

c. $h(x) = 2x^5 + 5x^4 - 10x^3 + 10x^2 - 5x + 1$

d. $j(t) = (1/11)(t^3 - 9t^2 + 27t - 32)$

0.4. Solve the equation for x using any convenient method:

a. $(x+3)^2 = 81$

b. $x^2 - 2x - 1 = 0$

c. $2x^2 + x - 4 = x^2 + x + 7$

d. $x^3 + 2x + 1 = x^3 - x^2 + 5$

0.5. Write an equation for the line:

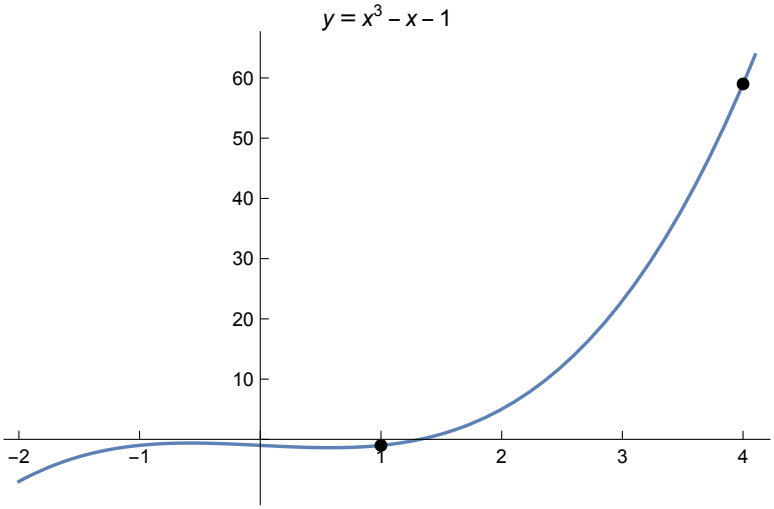
a. The line through $(0, 1/2)$ and $(4/3, 1)$

b. The line through $(5, 3\sqrt{2})$ with slope $5/8$.

The following problems are to be turned in. You may work alone or in a group of two; if you work in a pair (which is a good idea), submit just one solution with both names on it. You're welcome to ask for help or hints in office hours. If you ask the calculus tutors for help, they will expect to see your solutions to the skill problems on the previous page first.

Write a final draft of your solutions on clean paper (without ragged edges), leave me a generous amount of space around each problem to write comments, and staple multiple pages. Due at the beginning of class Friday, November 4th.

(10 pt) 1. The graph of the function $f(x) = x^3 - x - 1$ is shown, with two points marked at $x = 1$ and $x = 4$.



a. Compute the slope of the secant line connecting the two marked points on the graph, and write an equation for this line.

b. There is one value of x between 1 and 4 where the slope of the *tangent* line is the same as the slope of the secant line you just computed. Solve to find the x value where this happens, then write the equation for this tangent line.

c. Use Desmos or an equivalent graphing program to show the graph of the function f together with the secant line through the given points and the tangent line you have found. Use a range of values on the x- and y-axes similar to what I've used in the plot above, so that all the points of interest are clearly visible.

[Go, on, there's one more problem on the next page]

(10 pt) 2. Two water-propelled rockets with different designs are launched from the ground (height 0). Their heights (in cm) at time t (in sec) are given respectively by the functions

$$p_1(t) = (16/3)(27t^2 - t^3) \qquad \text{and} \qquad p_2(t) = (8/5)(105t^2 - 4t^3),$$

up until the time when they hit the ground again, at which time they stop moving entirely. Using centimeters isn't necessarily a great choice of units for the heights involved, but it keeps the coefficients in the polynomials reasonable.

a. There is one time during the flight when the two rockets are at the same height. Solve to find that time, and determine the height. Answer the question clearly with a sentence or two, and also show your solution method clearly.

b. There is also one time during the flight when the two rockets have the same velocity. Solve to find that time. Determine the velocity at that time (which should be the same for both rockets), and their two heights at that time (which will be different). As before, answer the question clearly with a few sentences. Also show a clear solution that supports your answer.

Note that the heights will be pretty large numbers, since they're expressed in centimeters, but they're realistic. You might want to convert to feet to check the plausibility of your answer.