For the following problems, let

$$f(x) = x^{12} \qquad \text{and} \qquad g(x) = x^3 - x$$

Find a simple formula for each of the following.

Think of the two functions as

$f(\square) = (\square)^{12}$ and $g(\square) = (\square)^3 - (\square)$.

a. $f(g(x))$ $= (x^3 - x)^{12}$

b. $g(f(x))$ $= (x^{12})^3 - x^{12}$.

c. $f'(x)$ $= 12x^{11}$

For the next few problems, it will be helpful to remember that this means

$f'(\square) = 12(\square)^{11}$

For ex.,

e. $f'(2x)$ $= 12(2x)^{11}$

f. $f'(x+z)$ $= 12(x+z)^{11}$

g. $f'(g(x))$ $= 12(x^3-x)^{11}$

But the next one is different. You're asked to take the derivative of a composition, and you need the chain rule.

h. $\frac{d}{dx}f(g(x))$ $= f'(g(x)) \cdot g'(x)$ - that's the chain rule

$= \underbrace{12(x^3-x)^{11}}_{\text{like part (g)}} \cdot \underbrace{(3x^2-1)}_{\text{an easy deriv. you can do with old rules.}}$

i. $\frac{d}{dx}f(2x))$ $= f'(2x) \cdot \frac{d}{dx}(2x)$ - that's the chain rule

$= 12(2x)^{11} \cdot 2$

or more simply, $24(2x)^{11}$ but not: $\cancel{48x^{11}}$

j. $(g \circ f)'(x)$

$= g'(f(x)) \cdot f'(x)$ according to the chain rule

$= \underbrace{[3 \cdot (x^{12})^2 - 1]}_{\text{this is } g'(f(x))} \cdot \underbrace{12x^{11}}_{\text{this is } f'(x)}$

This could be simplified or rearranged in various ways, but for now, it's fine as is.