Ida the Idealized Bicyclist is moving along a straight road with position function

$$f(t) = 6t + 4 \quad \text{meters at time } t \text{ seconds}$$

1a. Compute the following: $f(0) = \mathbf{4}$, $f(10) = \mathbf{64}$, $f(t+4) = \mathbf{6t + 24 + 4 = 6t + 28}$, and $f(t + \Delta t) = \mathbf{6(t + \Delta t) + 4 = 6t + 6\ \Delta t + 4}$.

b. Explain in words what the value you got for $f(10)$ means in terms of Ida the bicyclist.
At time $t = 10$ seconds, Ida is 64 meters away from the road position we set to be "0 m." One may also observe that, after ten seconds, Ida is 60 meters away from where she was at time 0. We got the number 60 from f(10)-f(0)=64-4=60.

c. And explain what the expression that you got for $f(t+4)$ means.
Four seconds after time t, Ida is (6t+28) meters away from the road position set to be 0. One may observe that the distance between where Ida was at a time t and where Ida was 4 seconds after time t is always 24 meters (for every t).

2a. Compute the net change in f over the interval $[2, 10]$, and the average rate of change in f over the same interval. Give the appropriate units for both answers.

Net change: f(10)-f(2)=64-16=48 m

Average rate of change:

$$\mathbf{\frac{f(10) - f(2)}{10 - 2} = \frac{64 - 16}{8} = \frac{48\ m}{8\ sec} = 6\ m/s.}$$

b. Repeat all of part (a) using the interval $[0, 20]$.

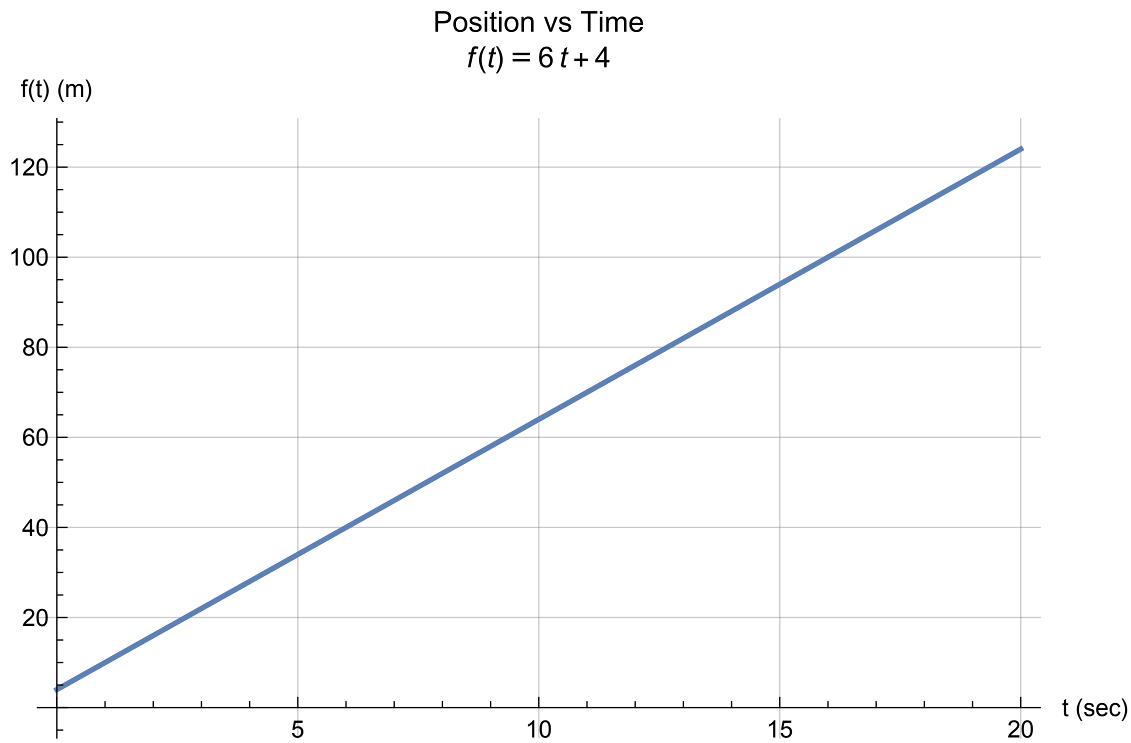
Net change over $[0, 20]$**: f(20)-f(0)=124 m - 4 m = 120 m**

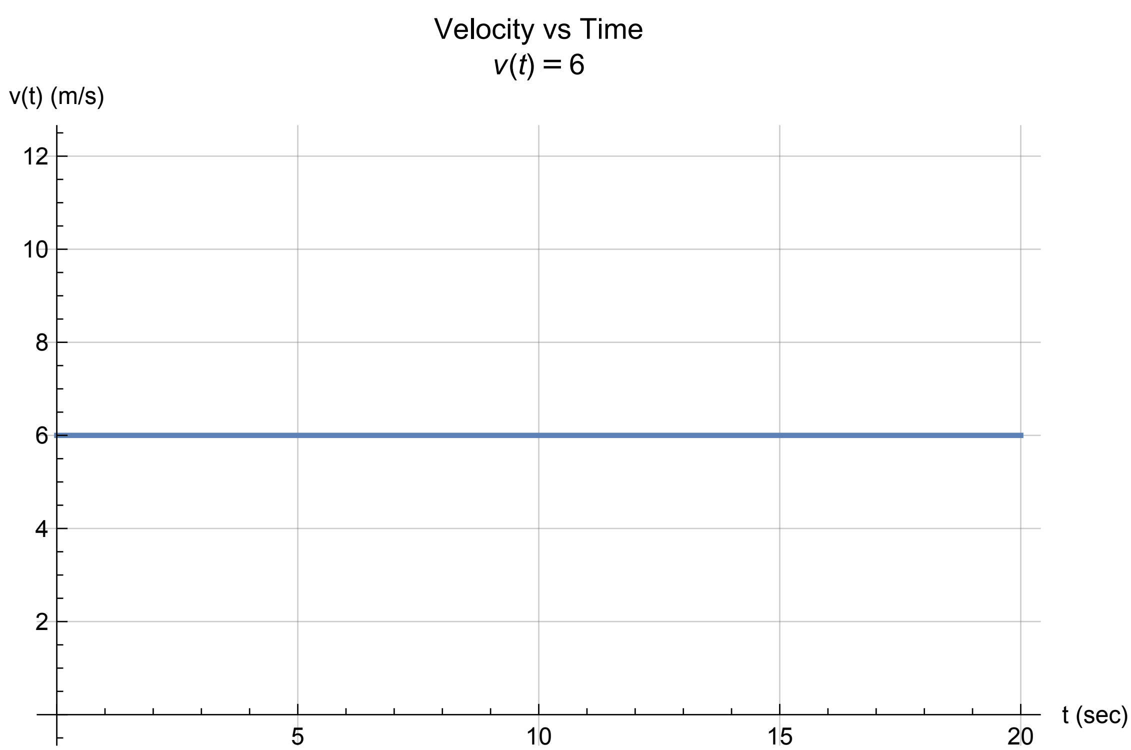
Average rate of change over $[0, 20]$**:**

$$\mathbf{\frac{f(20) - f(0)}{20 - 0} = \frac{124 - 4}{20 - 0} = \frac{120\ m}{20\ sec} = 6\ m/s.}$$

4. Let's call Ida's *velocity* function $v(t)$ (meters per second, at time t, where t is measured in seconds). Now, **I haven't given you a formula for $\mathbf{v(t)}$ yet**. Can you reason out what function this must be?
a. $\mathbf{v(t) = 6m/s}$, a constant function.

Recall from question 2 that the average rate of change in f over the interval $[2, 10]$ is 6 m/s, and the average rate of change in f over the interval $[0, 20]$ is 6 m/s. In fact, since f is a linear function with slope 6, the average rate of change in f over every interval is 6 m/s. It seems like Ida must be going at a constant rate of 6 m/s.


Position vs Time
$f(t) = 6t + 4$
f(t) (m)
120
100
80
60
40
20
5
10
15
20
t (sec)


Velocity vs Time
$v(t) = 6$
v(t) (m/s)
12
10
8
6
4
2
5
10
15
20
t (sec)

6. True/False: (f and v are the same functions from the problems above.)

a. The average rate of change in f is the same over every possible time interval.

True. We should expect this to be true since f is linear ("linear functions are the ones with the same rate of change over every possible interval"), but it is also something that we can check to be sure: If $[a, b]$ is any interval then the average rate of change in f over $[a, b]$ is

$$\frac{f(b) - f(a)}{b - a} = \frac{(6b + 4) - (6a + 4)}{b - a} = \frac{6(b - a)}{b - a} = 6 \text{ m/s},$$

no matter what a and b are.

b. The net change in f is the same over every possible time interval.

False. For example, you found a different net change over $[0, 20]$ than over $[2, 10]$.

c. The average rate of change in v is the same over every possible time interval.

True. Not only that, we can be more specific: the average rate of change in v is exactly 0 over every possible time interval, since v is constant: No matter what a and b are, the average rate of change in v over $[a, b]$ is $(v(b) - v(a))/(b - a) = (6 - 6)/(b - a) = 0$ m/s^2.

d. v is increasing over every interval.

False. Actually, v is *constant* over every interval. v never increases (or decreases) at all.

e. f is increasing over every interval

True. We can see this on the graph of f, or reason it out from the fact that the slope of f is positive 6 – going right on the graph always means going up as well.