This is a selective review for Test 4. It doesn't include every problem that could be on the test, and places more emphasis on definitions and statements of theorems than the actual test will. However, they are the definitions and theorems you'll need to know in order to understand and do well on the test. There is a more extensive review, including more computational problems, available on the course website.

Chain rule and related concepts

1. Use derivative rules to find $f'(x)$. Don't spend a lot of time simplifying once the differentiation steps are completed.
   (a) $f(x) = (x^2+1)^3$.
   (b) $f(x) = \dfrac{-7}{(2x-3)^2}$.
   (c) $f(x) = \sqrt[3]{(x^2-1)^2}$.
2. Find dy/dx given that $y^3 + y^2 - 5y - x^2 = -4$. (Hint: Differentiate both sides with respect to x, then solve for dy/dx).
3. Suppose that x and y are both differentiable functions of t, and they are related by the equation $y - x^2 = 3$. Find dy/dt when $x = 1$, given that $dx/dt = 2$ when $x = 1$.

Increase, decrease, maxes and mins

1. Complete the statement of Theorem 5.1 (The Extreme Value Theorem):

   If f is __________________ on ____________________________________

   then f has both ____
2. True/False:
   (a) If a function is continuous on $[-3, 3]$, then it must have a minimum on the interval.
   (b) The maximum of a function that is continuous on a closed interval can occur at only one value in the interval.
3. Complete the definition: Let f be a function which is defined at c. Then we say that c is a *critical number* of f if ____.
4. Let $f(x) = 2x^3 - 3x^2 - 36x + 14$.
   (a) What are the critical numbers of f?
   (b) Find all intervals (if any) on which f is increasing.
   (c) Classify the critical numbers (relative max/ relative min/ neither).
5. True/False: If 5 is a critical number of f, then f must have a relative minimum or relative maximum at $x = 5$.
6. True/False: If f has a relative maximum at $x = 2$, then 2 must be a critical number of f.

Concavity, inflection points, and the shape of graphs

1. Complete the definitions. Let f be differentiable on an open interval I.

   a. The graph of f is *concave up on I* if ______________________________.

   b. The graph of f is *concave down on I* if ______________________________.

2. Complete the statement of Theorem 5.7 (Test for Concavity):

   Let f be a function whose __________________________ exists on an open interval I.

   If ________________________________ then the graph of f is concave up on I.

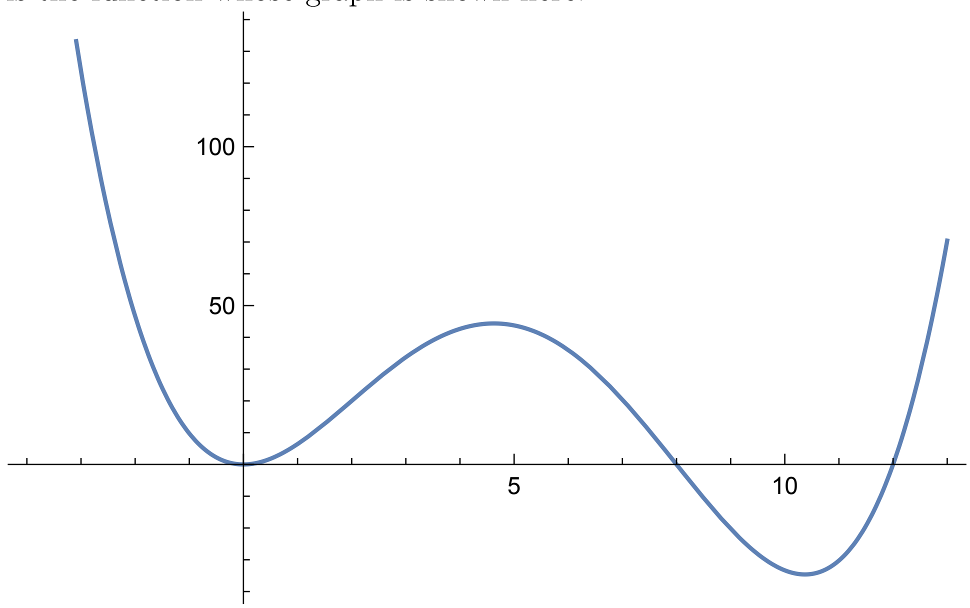
   If ________________________________ then the graph of f is concave down on I.

3. Suppose $f(x) = x^{11}$.

   (a) Determine the intervals on which the graph of f is concave up or down.

   (b) Does f have any inflection points? How many?

4. Suppose $g(x)$ is the function whose graph is shown here:



5. In the interval that we can see on the graph,

   (a) How many real zeros does $g(x)$ have?

   (b) How many real zeros does $g'(x)$ have?

   (c) How many real zeros does $g''(x)$ have?

   (d) How many critical numbers does $g(x)$ have?

   (e) How many relative maxima does $g(x)$ have?

   (f) How many relative minima does $g(x)$ have?

   (g) How many inflection points does $g(x)$ have?

   (h) If $g(x)$ is a polynomial, what is the smallest possible degree it could have?