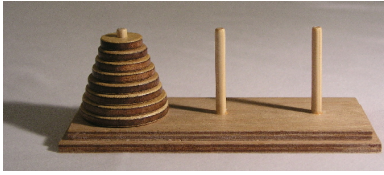


1. Consider the Tower of Hanoi puzzle.



- i.) Is it possible to solve this puzzle in $2^n - 1$ moves? Play this puzzle with your n paper disks, where $n = 1, 2, 3$, and 4.
 - ii.) Prove (using induction) that it is possible to solve this puzzle in $2^n - 1$ moves, or give a counterexample (find a number M where you need more than $2^M - 1$ to solve the puzzle).
2. Draw a complete graph of 5 vertices on the board. Draw a circle with 5 vertices and draw a star-shape using the vertices. Count the number of edges in this graph.
- I.) If the star has N vertices, how many edges are there? Write the answer as a closed-form formula.
 - II.) Prove your closed-form formula using induction.
3. The following even numbers can be written as the sum of two primes:

$$6 = 3 + 3$$

$$8 = 3 + 5$$

$$10 = 3 + 7 = 5 + 5$$

$$12 = 7 + 5$$

- a. Write each of the even numbers 50, 70, and 100 as the sum of two primes. Find as many ways as possible to write them as sums of two primes.
- b. Is it possible to write every even number (larger than 2) as the sum of two primes?
- c. Prove (using induction) that it is possible, or give a counterexample (find an even number $M > 2$ which cannot be written as the sum of two primes).