# MATH3094 WEEK 15 TH HANDWRITTEN HW 

REPLACE WITH YOUR NAME

List of positive definite graphs. Take 5 minutes to copy by hand the just first two columns of Table I (page 296-297 of Bjorner Brenti): the finite irreducible Coxeter systems. The first two columns are just the names $A_{n}, B_{n}, D_{n}, E_{6}, E_{7}, E_{8}, F_{4}, G_{2}, H_{3}, H_{4}, I_{2}(m)$ and their Coxeter graphs.

This table is also available on https://en.wikipedia.org/wiki/ Coxeter-Dynkin_diagram

## (A multiple of) Cartan matrices. See page 31 of Humphreys Sec

 2.3: For each Coxeter graph $\Gamma$ with vertex set $S$, we can define a symmetric $n \times n$ matrix $A=\left(a_{i, j}\right)$ by setting$$
a\left(s, s^{\prime}\right):=-\cos \frac{\pi}{m\left(s, s^{\prime}\right)}
$$

For example, the matrix for $I_{2}(6)$ is $\left(\begin{array}{cc}1 & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & 1\end{array}\right)$ and its determinant is $1-3 / 4=1 / 4>0$.

The principal minor of $A$ are the determinants of the submatrices of $A$ obtained by removing the last $k$ rows and columns $(0 \leq k<n)$. For example, the principal minors of the above matrix are 1 and $\frac{1}{4}$.
(1) For each of the Coxeter graphs $A_{2}, A_{3}, B_{2}, B_{3}, G_{2}$, write down the matrix $A$.
(2) Compute all the principal minors of the matrices $A$ for $A_{3}$ and $B_{3}$. Each matrix has three principal minors coming from the determinants of the $1 \times 1,2 \times 2$ submatrices and the determinant of $A$. Make sure all the principal minors are positive. You can use WolframAlpha or another tool to do or to check your determinant computation.

