## MATH3094 WEEK 15 TH HANDWRITTEN HW

## REPLACE WITH YOUR NAME

List of positive definite graphs. Take 5 minutes to copy by hand the just first two columns of Table I (page 296-297 of Bjorner Brenti): the finite irreducible Coxeter systems. The first two columns are just the names  $A_n$ ,  $B_n$ ,  $D_n$ ,  $E_6$ ,  $E_7$ ,  $E_8$ ,  $F_4$ ,  $G_2$ ,  $H_3$ ,  $H_4$ ,  $I_2(m)$  and their Coxeter graphs.

This table is also available on https://en.wikipedia.org/wiki/ Coxeter-Dynkin\_diagram

(A multiple of) Cartan matrices. See page 31 of Humphreys Sec 2.3: For each Coxeter graph  $\Gamma$  with vertex set S, we can define a symmetric  $n \times n$  matrix  $A = (a_{i,j})$  by setting

$$a(s,s') := -\cos\frac{\pi}{m(s,s')}.$$

For example, the matrix for  $I_2(6)$  is  $\begin{pmatrix} 1 & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & 1 \end{pmatrix}$  and its determinant is 1 - 3/4 = 1/4 > 0.

The principal minor of A are the determinants of the submatrices of A obtained by removing the last k rows and columns  $(0 \le k < n)$ . For

- example, the principal minors of the above matrix are 1 and <sup>1</sup>/<sub>4</sub>.
  (1) For each of the Coxeter graphs A<sub>2</sub>, A<sub>3</sub>, B<sub>2</sub>, B<sub>3</sub>, G<sub>2</sub>, write down the matrix A.
  - (2) Compute all the principal minors of the matrices A for  $A_3$  and  $B_3$ . Each matrix has three principal minors coming from the determinants of the  $1 \times 1$ ,  $2 \times 2$  submatrices and the determinant of A. Make sure all the principal minors are positive. You can use WolframAlpha or another tool to do or to check your determinant computation.