MATH3094 WEEK 12 HW (HANDWRITTEN IS OK)

REPLACE WITH YOUR NAME

Credit:

fill in
— Emily

Exercise 1. Reference: [Humphreys, Sections 1.6] and Week 11 class notes. Choose a simple system $\Delta = \{\alpha, \beta\}$ with $\alpha = (1, 0)$ and $\beta = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Its corresponding positive system is $\Pi = \{\alpha, \beta, \alpha + \beta\}$.

The following are the inversion sets for all elements of the reflection group.

(1) $N(id) = \emptyset$
(2) $N(\sigma_\alpha) = \{\alpha\}$
(3) $N(\sigma_\beta) = \{\beta\}$
(4) $N(\sigma_\alpha\sigma_\beta) = \{\beta, \alpha + \beta\}$
(5) $N(\sigma_\beta\sigma_\alpha) = \{\alpha, \alpha + \beta\}$ (computed during class)
(6) $N(\sigma_\alpha\sigma_\beta\sigma_\alpha) = \Pi$

Confirm the items labeled (2) and (6) using a combination of (a) the reflection formula (first page on Humphreys) and taking the usual dot product, (b) the permutation matrices, and (c) drawing the two-dimensional sketch.

Exercise 2. Reference: [Humphreys, Sections 1.6], Week 11 class notes, and Section 1.5 (page 20) of [Bjorner and Brenti].

(1) All 24 permutations of S_4 are listed in Figure 3.2 page 67 of [BB]. Pick two permutations π in S_4 (not computed in class) which are not adjacent transpositions $(k, k + 1)$, for example, you can pick [3421] and [4213] (window notations). For each π, please write down which row it is on. The bottom row (with the identity permutation) is row number 0.
(2) Compute $\ell(\pi)$ for each π (of the two you pick above).
(3) Write *all* shortest product of simple reflections for each π.
(4) Find $INV(\pi)$. Then verify that $inv(\pi) = \ell(\pi)$.

Misc. Approximately how much time did you spend on this homework?