MATH3094 WEEK 10 HW (HANDWRITTEN IS OK)

REPLACE WITH YOUR NAME

Credit: Write down everyone who helped you, including classmates who contributed to your thought process (either through sharing insights or through being a sounding board). Write down the textbook and other written sources you used as well.

> Required
> — Emily

Instruction.

> remove this instruction section when you are done.
> — Emily

You can complete this homework by hand or LaTeX. Either submit a physical copy (in class) or upload a PDF (if handwritten) or invite me to your Overleaf project.

Note: You are encouraged to post on Piazza or come to office hour.

Exercises. Reference: Humphreys Sections $1.2 + 2.9$, and Section 1.3.

(1) (Sec 1.2 and 2.9) If Φ is a root system (that is, satisfies (R1) and (R2)), then Φ^V is also a root system.
How to do this problem: this was done in class during Week 9 (with a few blanks to fill in with computation) when discussing Section 1.2 and Section 2.9.

(2) (Sec 1.2 and 2.9) Show that the action of W on the root lattice $L(\Phi)$ is stable, that is, if $v \in L(\Phi)$ then $\sigma_\alpha(v) \in L(\Phi)$ for all $\alpha \in \Phi$.
How to do this problem: this was done in class (when discussing Section 1.2 and Section 2.9).

(3) (Optional - will be on future Problem Set) (Sec 1.2 and 2.9) Show that if Φ is a crystallograhic root system then the action of W on the weight lattice $\hat{L}(\Phi)$ is stable, that is, if $v \in \hat{L}(\Phi)$ then $\sigma_\alpha(v) \in \hat{L}(\Phi)$ for all $\alpha \in \Phi$.

(4) (Sec 1.3) Let Π be a positive system of a root system Φ. Suppose D is a minimal subset subject to the requirement that each root in Π is a nonnegative linear combination of D. Prove that

$$\langle \alpha, \beta \rangle \le 0 \text{ for all pairs } \alpha \ne \beta \in D.$$

How to do this problem: This inequality statement is labeled (1) on [**?**, page 8]. The inequality is proven on [**?**, page 9]. Please read the proof many times until you understand it and

then rewrite the same proof here with more details. You can also read [**?**, page 34].

(5) Approximately how much time did you spend on this homework?

REFERENCES