MATH 3094 WEEK 8 PROBLEM SET BY YOURNAME

YOUR PREFERRED FIRST AND LAST NAME

Credit: Write down everyone who helped you, including classmates who contributed to your thought process (either through sharing insights or through being a sounding board). Write down Judson's textbook and other written sources you used as well.

Required
— Emily

Instruction.

Please remove this instruction section when you are done.
— Emily

Complete this problem set by LaTeX. Please send me an invite via Overleaf by entering my email address.

Note: If you are not sure how to do something, please post on Piazza or come to office hour.

1. From Judson Sec 11.3

Problem Set from Judson Sec 11.3 Exercises `http://abstract.ups.edu/aata/exercises-homomorph.html`, please complete *at least one* or both of Exercises 5 and 6 (and their lemmas).

(1) Lemma for Exercise 5. List all the subgroups of $(\mathbb{Z}_{24}, +)$.
(2) Exercise 5. Describe all the group homomorphisms from $(\mathbb{Z}_{24}, +)$ to $(\mathbb{Z}_{18}, +)$.
Hint: read Sec 11.1 Example 11.8 (`http://abstract.ups.edu/aata/section-group-homomorphisms.html`) or class notes.
Hint: a homomorphism must be of the form $z \mapsto kz$ where k is a positive integer.

(3) Lemma for Exercise 6. List all the subgroups of $(\mathbb{Z}, +)$.
(4) Exercise 6. Describe all the group homomorphisms from $(\mathbb{Z}, +)$ to $(\mathbb{Z}_{12}, +)$. Hint: read Sec 11.1 Example 11.8 (`http://abstract.ups.edu/aata/section-group-homomorphisms.html`) or class notes.

2. From Humphreys

(1) Prove that the transpositions are the only reflections belonging to S_n when S_n is viewed as a subgroup of the group $O(n, \mathbb{R})$ of orthogonal matrices.

Hint 1: See my proof (which is almost complete) from class notes

(2) Let L be the line spanned by the vector $\epsilon_1 + \epsilon_2 + \cdots + \epsilon_n \in \mathbb{R}^n$, that is,
$$L = \{(a, a, \ldots, a) | a \in \mathbb{R}\}.$$
Let $L^\perp$ denote the *orthogonal complement* of L, that is,
$$L^\perp = \{v \in \mathbb{R}^n | v.\ell = 0 \text{ for all } \ell \in L\}.$$
   (a) Describe $L^\perp$. Hint: see class notes or page 5 of Humphreys.
   (b) Convince yourself that S_n fixes pointwise the points in L. (You can leave this part blank.)
   (c) Prove that the only fixed points of S_n are the points of L. That is, if $v \in \mathbb{R}^n \setminus L$ then there is a permutation $\pi \in S_n$ such that $M_\pi(v) \neq v$.
   (d) Prove that the action of S_n on $L^\perp$ is *stable*, that is,
$$M_\pi(L^\perp) \subset L^\perp \text{for every } \pi \in S_n.$$
   That is, for any given $\pi \in S_n$, show that $M_\pi(v) \in L^\perp$ for all $v \in L^\perp$.

(3) Is $\sigma_{\epsilon_1} \circ \sigma_{\epsilon_2} = \sigma_{\epsilon_1 + \epsilon_2}$? Prove or disprove.

3. Miscellaneous

(1) Share your work (at least one problem) and thought process with at least one classmate. Ask them to share their thought process as well. Write down their names and briefly summarize your interactions. A virtual discussion via Piazza or email is fine if you don't have time to interact in person.
(2) Approximately how much time did you spend on this homework?