MATH 3094 WEEK 7 HW (HANDWRITTEN WORK IS OK)

REPLACE WITH YOUR NAME

Credit: Write down everyone who helped you, including classmates who contributed to your thought process (either through sharing insights or through being a sounding board). Write down Judson's textbook and other written sources you used as well.

Required
— Emily

Instruction.

remove this instruction section when you are done.
— Emily

You can complete this homework by hand or LaTeX. Either submit a physical copy (in class) or upload a PDF (if handwritten) or your Overleaf link on HuskyCT.

Note: You are encouraged to post on Piazza or come to office hour.

Exercises. From Judson Sec 11.3 Exercises `http://abstract.ups.edu/aata/exercises-homomorph.html`, please complete ...

i.) (Required) Find at least one classmate (a different person from two weeks ago), and share and discuss at least a couple homework exercises with them for at least a few minutes. Write down their name/s and briefly summarize your interaction with them (one to four complete sentences).

ii.) Exercise 1. Prove that $\det(AB) = \det(A)\,\det(B)$ for $A, B \in GL_2(\mathbb{R})$.
(Note: This shows that the determinant is a group homomorphism from $GL_2(\mathbb{R})$ (with matrix multiplication as the group operation) to $\mathbb{R}^*$, the nonzero real numbers, with multiplication as the group operation.)

iii.) Exercise 2. Consider the four maps ϕ between groups defined on Exercise 2 parts (a), (b), (c), (e).
- For each map, determine whether it is a homomorphism or not a homomorphism.
- For each map which is a homomorphism, what is the kernel?
- For each map which is not a homomorphism, explain why (give an example).

Date: deadline: Week 7 Tuesday, Oct 9, 2018, 3:30pm.

iv.) Exercise 4. Let $\phi : (\mathbb{Z}, +) \to (\mathbb{Z}, +)$ be the map given by $\phi(n) = 7n$ for $n \in \mathbb{Z}$. Find the kernel and the image of ϕ.

v.) (Not from Judson) Prove or disprove the following: if G is a cyclic group and H is a subgroup of G, then H is also a cyclic group.
Hint: You can study and paraphrase Section 2 of Conrad's subgroups of cyclic groups notes `http://www.math.uconn.edu/~kconrad/blurbs/grouptheory/cyclicgp.pdf`. Warning: many examples shown are for groups with multiplication as the group operation, unlike our problems.

vi.) (Only submit if you weren't in class on this day) We proved Exercise 18 in class. Practice proving this for an exam.

vii.) Exercise 19 part i. Given a homomorphism $\phi : G \to H$ define a relation $\sim$ on G by $a \sim b$ if $\phi(a) = \phi(b)$ for $a, b \in G$. Show that this relation is an equivalence relation.
Optional: describe the equivalence classes.

viii.) (Not from Judson) Let

$$S_n^B := \{\text{bijections } w : [\pm n] \to [\pm n] \text{ where } w(-a) = -w(a)\}.$$

Prove that this set S_n^B is closed under composition (using just the definition of the set).

Optional I: show also that $S_n^D := \{w \in S_n^B : neg(w(1), \ldots, w(n)) \equiv 0(\mod 2)\}$ is closed under composition.
Optional II: Show also that the set of affine permutations is closed under composition.

ix.) Do at least two of the following.
- Check that σ_α is an orthogonal transformation. Use the bilinearity of $<, >$.
- Check that σ_α sends α to $-\alpha$.
- Check that, if $-\beta \in H_\alpha$, then σ_α sends β to itself.
- Check that σ_α^2 is the identity map.

x.) Do Exercise 1 (Chapter 1) from Bjorner Brenti textbook, pg 22.

xi.) Approximately how much time did you spend on this homework?