MATH 3094 WEEK 4 PROBLEM SET

PREFERRED FIRST NAME AND LAST NAME

Credit: Write down everyone who helped you, including classmates who contributed to your proof-writing process (either through explaining their work or through being a sounding board during class discussion). Write down Judson's textbook and other written sources you used to write your proofs.

Instruction.

Please remove this instruction section when you are done.

— Emily

If you have a question for the instructor, you can write it in a box this way

— ReplaceWithYourName

When you are done writing, please click the 'share' button at the top of the screen to copy the Read & Edit link for your project. Please submit this Overleaf link on HuskyCT.

Reminder:

- You can google how to do certain things in LaTeX and use people's LaTeX help without attribution.
- Start every sentence with an English word. Capitalize all letters that should be capitalized. End each sentence with a period.

1. Discussing your work

Share your work (at least one problem) and thought process with at least one classmate. Ask them to share their thought process as well. Write down their names and briefly summarize your interactions.

2. List of Old Week 2 problems (all from Judson Chapter 5).

From this section, please submit the problems that you have not done yet.

— Emily

Hints are hidden as comments in the source code.

This is how to write comments in a smaller box

— Emily

This is how to write a comment on the side

— ReplaceWithYourName

Date: deadline: Week 4 Saturday, September 8, 2018.

Link to Judson's Exercises Chapter 5: `http://abstract.ups.edu/aata/exercises-permute.html`

I. Exercise 21. Let $\sigma \in S_n$ be not a cycle. Prove that σ can be written as the product of at most $n-2$ transpositions. Click here if you want to see a hint.

Proof. Suppose σ is a permutation in S_n that is not a cycle. Fill in with the rest of your proof. □

II. Exercise 22. If σ can be expressed as the product of an odd number of transpositions, show that any other product of transpositions equaling σ must also be odd. Click here if you want to see a hint.

Proof. Suppose σ can be expressed as the product of an odd number of transpositions. Fill in with the rest of your proof. □

III. Exercise 23. If σ is a cycle of odd length, prove that σ^2 is also a cycle. Click here if you are not sure how to begin.

Proof. Suppose σ is a cycle of odd length. Fill in with the rest of your proof. □

IV. Lemma for Exercise 26.

a. Write the transposition $(a\,b)$ as a finite product of

$$(12), (13), (14), \ldots, (1n).$$

Warm-up: First try writing a product for $a = 2$, $b = 5$.

b. Write the transposition $(a\,b)$ as a finite product of

$$(12), (23), (34), \ldots, (n-1, n).$$

Warm-up: First try writing a product for $a = 2$, $b = 5$.

c. Write the transposition $(a\,b)$ as a finite product of the two cycles (12) and $(123\ldots n)$.

Warm-up: First try writing a product for $a = 2$, $b = 5$. Click here to see a hint.

V. Exercise 26.

a. Prove that any permutation in S_n can be written as a finite product of $(12), (13), (14), \ldots, (1n)$.

b. Prove that any permutation in S_n can be written as a finite product of $(12), (23), (34), \ldots, (n-1, n)$.

c. Prove that any permutation in S_n can be written as a finite product of the two cycles (12) and $(123\ldots n)$.

Click here to see a hint.

Proof. Suppose σ is a permutation in S_n. Fill in with the rest of your proof. □

VI. Exercise 30. Let $\tau = (1, 2, 3, \ldots, k)$.

a. Prove that if σ is any permutation, then

$$\sigma\tau\sigma^{-1} = (\sigma(1), \sigma(2), \sigma(3), \ldots, \sigma(k))$$

Click here to see a hint.

Proof. Suppose σ is a permutation in S_n. Fill in with the rest of your proof. □

b. Let $\mu = (b_1, b_2, \ldots, b_k)$ be a cycle of length k. Find a permutation σ such that $\sigma\tau\sigma^{-1} = \mu$.

3. List of new problems Week 4 (All from Judson Chapter 3)

Please submit two or more problems from this list. You can remove or comment out the problems you omit.

— Emily

Link to Judson Chapter 3 Exercises: `http://abstract.ups.edu/aata/exercises-groups.html`

I Exercise 7. Let $S = \mathbb{R}\backslash\{-1\}$. Define a binary operation on S by $a \star b = a + b + ab$. Prove that $(S, \star)$ is an Abelian group. That is, after checking that the set S is closed under $\star$, you only need to check that (i) $\star$ is associative, (ii) S contains an identity , (iii) every element has an inverse, and (iv) every pair of elements commute.

(what is it?)
— Emily

II Exercise 28. Prove the second half of Proposition 3.21.

III Exercise 31. If $a^2 = e$ for all elements a in a group G, then G must be Abelian. Click here for a hint.

IV Exercise 32. Show that if G is a finite group of even order, then there is an $a \in G$ such that a is not the identity and $a^2 = e$.

V Exercise 34. Find all the subgroups of $\mathbb{Z}\backslash 3\mathbb{Z} \times \mathbb{Z}\backslash 3\mathbb{Z}$. Use this information to show that $\mathbb{Z}\backslash 3\mathbb{Z} \times \mathbb{Z}\backslash 3\mathbb{Z}$ is not the same group as $\mathbb{Z}\backslash 9\mathbb{Z}$. Click here for an answer to a similar problem.

VI Exercise 40. Let G consist of 2×2 matrices of the form

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

where $\theta \in \mathbb{R}$. Prove that G is a subgroup of $SL_2(\mathbb{R})$, the group of all 2×2 matrices with determinant 1. You can take for granted that the matrix multiplication is associative and that the identity matrix is the identity of this binary operation. You only need to show that G is closed under matrix multiplication and that each element has an inverse. Hint: look up trig identities.

VII Exercise 48. Let G be a group and g a fixed element of G. Show that

$$Z(G) = \{x \in G : gx = xg \text{ for all } g \in G\}$$

is a subgroup of G.

(You can use one of the subgroup theorems in `http://abstract.ups.edu/aata/section-subgroups.html#groups-subsection-subgroup-theorems`).

Note: $Z(G)$ is called the *center* of G.

VIII Exercise 51. If $xy = x^{-1}y^{-1}$ for all x and y in G, prove that G must be abelian. Click here for a hint.

IX Exercise 53. Let G be a group and H a subgroup of G. Show that

$$C(H) = \{g \in G : gh = hg \text{ for all } h \in G\}$$

is a subgroup of G.

(You can use one of the subgroup theorems in `http://abstract.ups.edu/aata/section-subgroups.html#groups-subsection-subgroup-theorems`).

Note: $C(G)$ is called the *centralizer* of G.

X Exercise 54. Let G be a group and let H be a subgroup of G. If $g \in G$, define

$$gHg^{-1} := \{ghg\}.$$

Show that gHg^{-1} is a subgroup of G. (You can use one of the subgroup theorems in `http://abstract.ups.edu/aata/section-subgroups.html#groups-subsection-subgroup-theorems`).

Note: This subgroup is called the centralizer of H in G.

Please write approximately how much time you spend on this problem set and include comments, if you have any: