MATH 3094 WEEK 2 PROBLEM SET (TYPED IN LaTeX)

PREFERRED FIRST NAME AND LAST NAME

Credit: Write down everyone who helped you, including classmates who contributed to your proof-writing process (either through explaining their work or through being a sounding board during class discussion). Write down Judson's textbook and other written sources you used to write your proofs.

Instruction. Please remove this instruction section when you are done.

When you are done writing, please click the 'share' button at the top of the screen to copy the Read & Edit link for your project. Please submit this Overleaf link on HuskyCT. If you don't have access to the course HuskyCT, please create a private post to share your Overleaf link on Piazza.

Reminder:

- You can google how to do certain things in LaTeX and use people's LaTeX help without attribution.
- Start every sentence with an English word. Capitalize all letters that should be capitalized. End each sentence with a period.

List of problems (all from Judson Chapter 5). For full credit, submit *a minimum of 3 problems*. Hints are hidden as comments in the source code. If you want more challenge, you can first erase all hints so that you won't accidentally look at them. Link to Judson's Exercises Chapter 5: `http://abstract.ups.edu/aata/exercises-permute.html`

I. Exercise 21. Let $\sigma \in S_n$ be not a cycle. Prove that σ can be written as the product of at most $n-2$ transpositions. Click here if you want to see a hint.

Proof. Suppose σ is a permutation in S_n that is not a cycle. Fill in with the rest of your proof. □

II. Exercise 22. If σ can be expressed as the product of an odd number of transpositions, show that any other product of transpositions equaling σ must also be odd. Click here if you want to see a hint.

Date: deadline: Week 2 Saturday, September 8, 2018, 5pm.

Proof. Suppose σ can be expressed as the product of an odd number of transpositions. Fill in with the rest of your proof. □

III. Exercise 23. If σ is a cycle of odd length, prove that σ^2 is also a cycle. Click here if you are not sure how to begin.

Proof. Suppose σ is a cycle of odd length. Fill in with the rest of your proof. □

IV. Lemma for Exercise 26. Do at least one of (a), (b), or (c). You might find the last option most interesting.

a. Write the transposition $(a\,b)$ as a finite product of

$$(12), (13), (14), \ldots, (1n).$$

Warm-up: First try writing a product for $a = 2$, $b = 5$.

b. Write the transposition $(a\,b)$ as a finite product of

$$(12), (23), (34), \ldots, (n-1, n).$$

Warm-up: First try writing a product for $a = 2$, $b = 5$.

c. Write the transposition $(a\,b)$ as a finite product of the two cycles

$$(12) \text{ and } (123\ldots n).$$

Warm-up: First try writing a product for $a = 2$, $b = 5$. Click here to see a hint.

V. Exercise 26. Do either (a), (b), or (c).

a. Prove that any permutation in S_n can be written as a finite product of $(12), (13), (14), \ldots, (1n)$.

b. Prove that any permutation in S_n can be written as a finite product of $(12), (23), (34), \ldots, (n-1, n)$.

c. Prove that any permutation in S_n can be written as a finite product of the two cycles (12) and $(123\ldots n)$.

Click here to see a hint.

Proof. Suppose σ is a permutation in S_n. Fill in with the rest of your proof. □

VI. Exercise 30. Let $\tau = (1, 2, 3, \ldots, k)$.

a. Prove that if σ is any permutation, then

$$\sigma\tau\sigma^{-1} = (\sigma(1), \sigma(2), \sigma(3), \ldots, \sigma(k))$$

Click here to see a hint.

Proof. Suppose σ is a permutation in S_n. Fill in with the rest of your proof. □

b. Let $\mu = (b_1, b_2, \ldots, b_k)$ be a cycle of length k. Find a permutation σ such that $\sigma\tau\sigma^{-1} = \mu$.

Please write approximately how much time you spend on this problem set and include comments, if you have any: