MATH3250 COMBINATORICS PROBLEMS WEEK 4

For the sake of learning the tools of Chapter 3, when you work on Problems 1 - 5 below, please model your explanations after the the proofs for Thm 3.6, Examples 3.7, 3.11, 3.12 in Sec 3.2; Thm 3.16 in Sec 3.3.

1. Functions

Let n be a positive integer.

(1) How many injective functions are there from $[n]$ to $[n]$?

(2) How many functions are there from $[n]$ to $[n]$ that are not injective?

Hint: Combine Thm 3.6 (the number of k-digit strings over an n-element alphabet) and the solution to Example 3.11 (the bijection from the collection of subsets of $[n]$ onto the collection of n-digit binary strings.)

2. Interns

A local company has 8 interns from UConn and 12 interns from Eastern Connecticut State University (ECSU). The company would like to form a service committee consisting of the interns.

(1) How many ways are there to form this committee consisting of 2 UConn interns and 3 ECSU interns?

(2) How many ways are there to form a 5-people committee that contains *at least* one UConn student and one ECSU student?

3. Polygon diagonals

Let $n \geq 4$. Consider a convex n-gon that is drawn in such a way that no three diagonals intersect in one point. How many intersection points do the diagonals have? (For example, if you draw a pentagon, there are five diagonals and five crossings.) Prove this.

Hints: Whenever two diagonals cross, it counts as one intersection point. Draw some examples for $n = 4, 5$. Can you turn this into the problem of choosing k-subsets of $[n]$, as in Section 3.3 Choice Problems?

4. Rooks

We would like to place n rooks on an $n \times n$ chess board in such a way that *no rook can attack*. In chess, a rook is only allowed to move horizontally or vertically, through any number of unoccupied squares. Please see `https://en.wikipedia.org/wiki/Rook_(chess)` for a demonstration. In how many ways can we do this?

5. Combinatorics class

A Combinatorics class consists of n freshmen, n sophomores, and n seniors. For example, imagine a class with 12 students, so $n = 4$.

(1) The students are to form n presentation groups of three people each. How many ways can they do this if each group must contain a freshman, a sophomore and a senior?

(2) The n seniors are to form a circle. Two circle arrangements are considered identical if each person has the same left neighbor in the circles. How many ways can the n seniors to form a circle?

Hint: Imagine breaking the circle so that the seniors now form a straight line.