

## MATH3250 COMBINATORICS PROBLEMS WEEK 3

### 1. DIVISIBLE BY ELEVEN

Prove that a positive integer with digits  $a_1, a_2, \dots, a_n$  is divisible by 11 if and only if  $a_1 - a_2 + a_3 - \dots + (-1)^{n-1}a_n$  is divisible by 11.

*Proof.* Insert proof □

### 2. 6 DIGIT NUMBERS

- a) How many 6 digit numbers are there (leading zeros, e.g. 001223 not allowed)?
- b) How many of these are even?
- c) How many 6 digit numbers are there with exactly one 7?
- d) How many 6 digit numbers are there that are the same forward and backwards (e.g., 890098)?

*Proof.* Insert answers and explanations □

### 3. BAA, ABA, AAB

How many 3 digit positive integers contain two (but not three) digits? Example of such digits are 122, 343, 660.

### 4. SANDWICH SHOP

A sandwich shop has 4 protein options. It also has 6 veggies: lettuce, sprouts, carrots, onion, tomato and pickles. It carries 5 sauces: mustard, catsup, mayo, sirachi, and vinegar. How many sandwiches can be made from one protein, one veggie and AT MOST one sauce?

*Proof.* Insert answer and a brief reasoning. □

### 5. CONNECTICUT

Compute the number of ways to create a list (of size 11) of the letters of the word CONNECTICUT.

*Proof.* Insert answer and a brief reasoning. □

### 6. ALTERNATING PARITY

- a) Warm-up: In how many ways can the elements of  $[3]$  be permuted so that the sum of every two consecutive elements in the permutation is odd? In how many ways can the elements of  $[4]$  be permuted so that the sum of every two consecutive elements in the permutation is odd?
- b) (Optional) Compute this for  $[5]$  as well.
- c) In how many ways can the elements of  $[n]$  be permuted so that the sum of every two consecutive elements in the permutation is odd?

### 7. WRITE YOUR OWN PROBLEM

Please write your own problem and solve it using theorems or concepts from Section 3.1 of Bona. (Student's problems may be chosen for future exams' questions). Test the difficulty level of your problem by sharing it with another math student.