MATH3250 COMBINATORICS READING HW 9

Instruction. Please submit all sections. Because you will be doing a lot of arithmetic and sketching, it might be more convenient to do this homework by hand. Use Bóna's "A Walk through Combinatorics" textbook.

Review of Sec 8.1.2 The Product Formula

(If you understood the lecture during class on Thursday, March 5, then this part is optional) Write down the problem and solution to Example 8.8 page 172. Explain the computation details skipped by the textbook.

1. Lucas and other Fibonacci/Pingala-like numbers

(This is another example related to Section 8.1.1: Recurrence relations and generating functions)
Let $a_1 = 1$, $a_2 = 3$, and $a_n = a_{n-1} + a_{n-2}$ for all $n \geq 1$.

(1) Compute a simple explicit formula for an ordinary generating function of a_n.
(2) Compute a closed-form (no summation) formula for a_n.

Note: If you are not sure how to do the problems, you can follow the first ten minutes of this Jim Fowler's video for computing a formula for the Fibonacci/Pingala numbers youtube.com/watch?v=CR-nmp97Ayo. Your final answer will be slightly different because your initial values are different, but you can follow his methods exactly.

2. Catalan numbers from the Internet

a. Pick two types of Catalan objects. For each type, draw the fourteen objects for $n = 4$ (so you will draw a total of 28 pictures). You can pick the ones introduced during class, or you can pick two objects from this Catalan numbers page http://mathshistory.st-andrews.ac.uk/Miscellaneous/CatalanNumbers/catalan.html. The Wikipedia entry for "Catalan number" also has a lot of nice pictures of some Catalan objects.
b. Spend about fifteen minutes reading a few pages linked from this Catalan numbers page, by Igor Pak: https://www.math.ucla.edu/ pak/lectures/Cat/pakcat.htm. Then write down two new facts about the Catalan numbers that you learned from these pages.
c. Attempt to find a bijection between two Catalan objects. If you can't, then read and explain one of the bijections given in these slides by Richard Stanley (just do ctrl+F or Apple-F for the word "bijection"): https://math.mit.edu/ rstan/transparencies/china.pdf

3. Catalan numbers: Tuples

We say that a tuple $(z_0 = 1, z_1, z_2, \ldots, z_{n-1}, z_n = 1)$ of positive integers is *admissible* if, for all $k \in \{1, \ldots, n-1\}$, the integer x_k divides $(x_{k-1} + x_{k+1})$. For example, there are exactly five admissible tuples for $n = 3$,

$$(1, 1, 1, 1),\ (1, 1, 2, 1),\ (1, 2, 1, 1),\ (1, 2, 3, 1), \text{ and } (1, 3, 2, 1).$$

a. For fifteen minutes, write down as many of the fourteen admissible tuples as you can for $n = 4$. It's OK if you don't find all fourteen.
b. Attempt to find a bijection between these admissible tuples and other Catalan objects you wrote about earlier (triangulations, trees, etc). Try googling. It's OK if you don't succeed.

4. Catalan numbers Sec 8.1.2.1

i. Go to Sec 8.1.2.1 Catalan numbers (pg 174–176). Read the pages a few times, until they make sense.
ii. Write down your notes (from reading the pages). Include the computation needed to get to Equation (8.14) and the formula $c_n = \frac{\binom{2n}{n}}{n+1}$ which were skipped by the book.

5. Survey

i. Approximately how much time did you spend on this homework?
ii. Questions or comments?