MATH3250 COMBINATORICS READING HW 14

Instruction.

- Do all sections. Submit your homework by email (subject: Math3250 Combinatorics Reading HW 14). Either type or hand-write your work (use a scanner app to convert to PDF)
- Ref: textbook *Combinatorics and Graph Theory* by Harris, Hirst, and Mossinghoff (HHM) Sec 1.2 and Bóna's "A Walk through Combinatorics" textbook, Chapter 9

1. Watch or read: HHM Sec 1.2 Distance in graphs

Do one of the following:

i. Finish watching lecture video of Sec 1.2 Distance of graphs
ii. Finish reading only the parts highlighted in color lecture notes for Sec 1.2 video

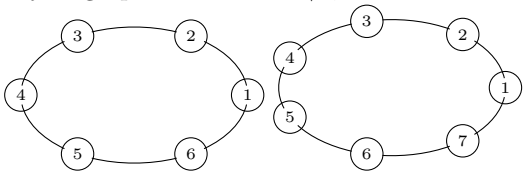
Write down what you did. If you watched the video, please specify (Kaltura/YouTube) and type of device.

2. Exercises (complete all)

Write down the definition of *adjacency matrix* and *distance matrix*. Give the adjacency matrix and the distance matrix of each of the following families of graphs:

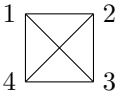
a.) the path graph P_n, where the vertices are labeled from one end of the path to the other.

b.) the cycle graph C_{2k} and C_{2k+1}, where the vertices are labeled consecutively around the cycle, *e.g.*



c.) the complete bipartite graph $K_{m,n}$, where the vertices in the first partite set are labeled $1, 2, \ldots, m$.
d.) the complete graph K_n, any labeling.

e.) Without computing the matrix directly, find A^3 where A is the adjacency matrix of the complete graph K_4 (shown below). Use Theorem 1.7 in HHM. Recall that $A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ since there is an edge between every pair of vertices.

$K_4 =$ 

f.) If A is the adjacency matrix for a graph G, show that the (j, j) entry of A^2 is the degree of v_j. (Previously there was a typo.)

g.) Find the ordinary generating function for the number of edges of the complete graph K_n, $n \geq 1$.
h.) Find the exponential generating function for the number of edges of the complete graph K_n, $n \geq 1$.

3. Presentations

- Pick several exercises (of out the eight listed above), and prepare to explain them during class meeting.
- Make sure to tell me your preferences ahead of time (to save time)

4. Last Section

Do one of the following:

(1) Read about 1/2 of the blog post math3ma.com/blog/matrices-probability-graphs by T.-D. Bradley which explains how every $m \times n$ matrix corresponds to a weighted bipartite graph, and matrix multiplication is the same as gluing two graphs and traveling along paths. Then mimic the graph-gluing process that the author did for matrix multiplication MN, but for the following matrices:

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 4 \end{bmatrix}, N = \begin{bmatrix} 5 \\ 0 \\ -1 \end{bmatrix}.$$

(2) Read *one* of the following four passages from HHM Sec 1.2.3 (p. 26–30): Acquaintance Graph / Hollywood Graph/ Mathematical Collaboration graph / Small World Networks. Then briefly summarize the main idea.