MATH3250 COMBINATORICS READING HW 13

Instruction.

- Do all exercises. Submit your homework by email (subject: Math3250 Combinatorics Reading HW 13).
- Your answers don't require math symbols beyond letters and numbers, so you can type your answers in the body of an email or in a word editor.
- You can also complete by hand (then scan using your smart phone to produce a PDF file) or use LaTeX.
- Ref: textbook *Combinatorics and Graph Theory* by Harris, Hirst, and Mossinghoff (HHM) Sec 1.1.3, 1.2.1 and Bóna's "A Walk through Combinatorics" textbook, Chapter 9

1. Watch or read: HHM Sec 1.1.3 Special types of graphs

Do one of the following:

i. Watch the lecture video of Sec 1.1.3 Special types of graphs
ii. Read only the parts highlighted in color lecture notes for Sec 1.1.3 video

- Write down which option you did. If you watched the video, specify Kaltura/YouTube and type of device.
- I edited this video to play at higher speed. Should I keep the videos at the original (slower) speed?

2. Exercises related to HHM Sec 1.1.3 Special types of graphs

a. (For the sake of practice) Give a proof for Claim 2 in the Theorem 1.3: for a vertex v of G, no two vertices in the set

$$Y := \{x \in V(G) \mid \text{a shortest part from } x \text{ to } v \text{ has odd length}\}$$

are adjacent.

b. (i) What down the definition of a regular graph. (ii) Prove: If the complete bipartite $K_{r,s}$ is regular, then $r = s$.

c. Let G be a connected graph with 22 edges. If G is *regular*, how many vertices can G have? See Bona Ch 9: Exercise 12, p. 220 for a sketch of a solution.

d. Show that the following statement is false (by providing a counterexample): Two graphs which have the same degree sequence must be isomorphic. See HHM Sec 1.1.3, p. 17: Exercise 9 or 10, or Bona Ch 9: Exercise 21.

e. HHM Sec 1.1.3, p.17: Exercise 10 (isomorphic/non-isomorphic)

f. Prove that there are more than 6600 pairwise non-isomorphic graphs with vertex set $\{1, 2, 3, 4, 5, 6, 7, 8\}$. See Bona Ch 9: Exercise 9, p. 220 for a sketch of a solution.

3. Watch or read: HHM Sec 1.2.1 Distance in graphs: radius and diameter

Do one of the following:

i. Watch the lecture video of Sec 1.2 Distance of graphs, only up to the definition of radius and diameter.
ii. Read only the parts highlighted in color lecture notes for Sec 1.2 video up to the definition of radius and diameter.

- Write down which option you did. If you watched the video, specify Kaltura/YouTube and type of device.
- I edited the video to play at higher speed. Should I keep the video at the original (slower) speed?

4. Radius and diameter exercises, from Sec 1.2.1

Write the definition of radius & diameter of a graph. Then find the radius and diameter of the following graphs:

A. the path graph P_{2k} and P_{2k+1}
B. the cycle graph C_{2k+1} and C_{2k+1}
C. the complete bipartite graph $K_{m,n}$
D. the complete graph K_n

5. Presentations

- Pick several exercises (of out the ten listed above), and prepare to explain them during class meeting.
- Make sure to tell me your preferences ahead of time (to save time)

It's fine (and maybe better for you) if you pick questions that you don't feel 100% about.