MATH3250 COMBINATORICS READING HW 11

Instruction.
- Submit your homework by email (subject line: Math3250 Combinatorics Reading HW 11).
- You can complete by hand (then scan using your smart phone to produce a PDF file) or type the homework. If you do this by hand, please use a pen or dark pencil so that the scan is readable.
- Use Bóna's "A Walk through Combinatorics" textbook.

1. Sec 8.2.1 Exponential Generating Functions lecture

Do one of the following:

i. Watch lecture video (12 minutes) of Sec 8.2.1
ii. Read lecture notes for Sec 8.2.1 video
iii. Read Sec 8.2.1 of the book (pg 180-182)

Write down which option you did. If you watched the video, please specify which platform (Kaltura or YouTube) and what type of device.

2. Sec 8.2.1 Exponential Gen. Functions: Solve Example 8.19

Explain in details the solution of Example 8.19 (pg 181). Be as detailed as my solution for Example 8.17.

3. Watch or read: Sec 8.2.2 Products of Exponential Generating Functions

Do one of the following:

i. Watch lecture video (15 minutes) of Sec 8.2.2
ii. Read lecture notes for Sec 8.2.2 video
iii. Read Sec 8.2.2 up to the first example.

Write down which option you did. If you watched the video, please specify which platform (Kaltura or YouTube) and what type of device.

4. Exercise: Sec 8.2.2 Products of Exponential Gen. Functions

A football coach has n players to work with at today's practice. First the coach splits the players into two *non-empty* groups, and then the coach puts the members of each group in a line. In how many different ways can all this happen?

(First, attempt to solve this following the same method in solving Example 8.22 explained in the video/ lecture notes. Go to the next page to see a sketch of the solution.)

5. Survey

i. Approximately how much time did you spend on this homework?
ii. You are encouraged to communicate with your classmates. Write down the resources (for example, Bóna's textbook or a math.stackexchange.com/ page) you referenced and the people that you talked with.
iii. Questions or comments?

Step-by-step outline for solving Problem 4:

- Let $A(x)$ be the *exponential* generating function (EGF) enumerating the number of ways for (nonzero) players to line up.
- Explain why $A(x) = \frac{x}{1-x}$.
- Let c_n be the number of ways for the coach to first splits the players into two non-empty groups then put the players of each group in a line. Let $C(x) := \sum_{n=0}^{n} c_n \frac{x^n}{n!}$. Use the Product formula for exponential generating functions to show that $C(x) = \frac{x^2}{(1-x)^2}$.
- Start with the geometric series then differentiate to show that $C(x) = \frac{x^2}{(1-x)^2} = \sum_{n=2}^{\infty}(n-1)x^n$.
- Apply the same method as in Example 8.17 and Example 8.22 to conclude that $c_n = n!(n-1)$ for $n \geq 2$.