

MATH3250 COMBINATORICS READING HW 1

Instruction. The following problems will be presented during class on Thursday, Jan 23. If you have attempted the problem before class (even if you don't have complete solution), you can volunteer to present the problems in a group of size 1 or 2. You can also volunteer to earn audience participation that day instead.

Problems 2, 3, and 4 will be graded based on effort.

Please use the Pigeon-Hole Principle (PHP) to solve all problems. Use the first chapter of Bona's textbook: www.worldscientific.com/doi/pdf/10.1142/9789813148857_0001

1. EXAMPLE 1.3 FROM BONA

A chess tournament has n participants, and any two players play exactly one game against each other. Is it true that in any given point of time, there are two players who have finished the same number of games?

- (1) First, attempt the following problem on your own or with other students (without looking up the textbook's solution)
- (2) Read the solution given under Example 1.3.
- (3) What play the role of "boxes" in this problem?
- (4) What play the role of "balls" in this problem?
- (5) Use the book's solution to write down a solution in your own words. It should be wordier than the book's solution. Try to first write it without looking at the book's solution.

2. EIGHT INTEGERS

Given eight distinct integers, prove that there are two integers whose difference is divisible by seven.

- (1) What play the role of "boxes" in this problem?
- (2) What play the role of "balls" in this problem?

3. SIX POSITIVE INTEGERS

A student wrote six distinct positive integers on the board, and pointed out that none of them had a prime factor larger than 10.

- (1) Prove that there are two integers on the board that have a common prime divisor.
- (2) Could you make the same conclusion if in the first sentence we replaced "six" by "five"?
- (3) What play the role of "boxes" in this problem?
- (4) What play the role of "balls" in this problem?

4. POLYNOMIAL DEGREES

The product of five given polynomials is a polynomial of degree 21. Prove that we can choose two of those polynomials so that the degree of their product is at least nine.

5. SURVEY

- i. Approximately how much time did you spend on this homework?
- ii. Any questions so far?