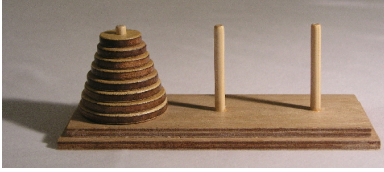
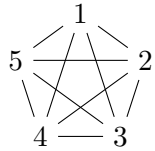


1. Consider the Tower of Hanoi puzzle.



- i.) Is it possible to solve this puzzle in  $2^n - 1$  moves? Play this puzzle with your  $n$  paper disks, where  $n = 1, 2, 3,$  and  $4$ .
- ii.) Prove (using induction) that it is possible to solve this puzzle in  $2^n - 1$  moves, or give a counterexample (find a number  $M$  where you need more than  $2^M - 1$  to solve the puzzle).
2. On the board, please draw the complete graph on 5 vertices



Count the number of edges in this graph.

- I.) How many edges are there for the complete graph on  $N$  vertices? Write the answer as an explicit (closed-form) formula.
- II.) Prove your closed-form formula using induction.
3. The following even numbers can be written as the sum of two primes:

$$6 = 3 + 3$$

$$8 = 3 + 5$$

$$10 = 3 + 7 = 5 + 5$$

$$12 = 7 + 5$$

- a. Write each of the even numbers 50, 70, and 100 as the sum of two primes. Find as many ways as possible to write them as sums of two primes.
- b. Is it possible to write every even number (larger than 2) as the sum of two primes?
- c. Prove (using induction) that it is possible, or give a counterexample (find an even number  $M > 2$  which cannot be written as the sum of two primes).