MATH3250 COMBINATORICS SAMPLE PROBLEM SET

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Note: In this class, we will use $[n]$ to denote the set $\{1, 2, \dots, n\}$.

1. The number of all subsets (Theorem 2.4 page 27)

For all positive integers n, the number of all subsets of $[n]$ is 2^n.

Proof (by induction). For $n = 1$, the statement is true as $[1] = \{1\}$ has two subsets, the empty set, and $\{1\}$.

Now let k be a positive integer, and assume that the statement is true for $n = k$. We divide the subset of $[k+1]$ into two classes: there will be those subsets that do not contain the element $k+1$, and there will be those that do. Those that do not contain $k+1$ are also subsets of $[k]$, so by the induction hypothesis their number is 2^k. Those that contain $k+1$ consist of $k+1$ and a subset of $[k]$. However, that subset of $[k]$ can be any of the 2^k subsets of $[k]$, so the number of these subsets of $[k+1]$ is once more 2^k. So altogether, $[k+1]$ has $2^k + 2^k = 2^{k+1}$ subsets, and the statement is proven. $\square$