MATH3250 COMBINATORICS WEEK2 PROBLEM SET

YOUR PREFERRED FIRST AND LAST NAME

Credit: Write down everyone who helped you, including classmates who contributed to your thought process (either through sharing insights or through being a sounding board). Write down Bona's textbook and other written sources you used as well.

Instruction. The exact problems will be announced on Monday during class. Please send me an invite via Overleaf by entering my UConn email address.

Note: If you are not sure how to do something, please post on Piazza or come to office hour.

Please remove this instruction section when you are done.

1. Six distinct integers

A student wrote six distinct positive integers on the board, and pointed out that none of them had a prime factor larger than 10.

(1) Prove that there are two integers on the board that have a common prime divisor.

Proof. Insert proof □

(2) Could you make the same conclusion if in the first sentence we replaced 'six' by 'five'? Explain.

Proof. Insert proof □

2. Wednesdays

The month of January 2019 has five Wednesdays. (Optional: How many months in 2019 contain five Wednesdays?) For any given year, use the Pigeon-hole Principle to determine the possible number of months that contain five Wednesdays. Click here for hints:

Proof. Insert proof □

3. Soccer Team

A soccer team scored a total of 40 goals this season. Nine players scored at least one of those goals. Prove that there are two players among those nine who scored the same number of goals.

Proof. □

4. Five real numbers whose sum is 100

Let's say I give you a mystery set of five positive real numbers whose sum is 100. Prove that there are two numbers among them whose difference is at most 10.

Proof. □

5. Swimming

(1) In the month of June, Mr. Consistent went swimming 26 times, though he never went more than once on the same day. Is it true that there were six consecutive days that he went swimming?
(2) Same as part (1), but for the month of July instead of June.

6. Polynomial degrees

The product of five given polynomials is a polynomial of degree 21. Prove that we can choose two of those polynomials so that the degree of their product is at least nine.

Click here for a hint:

Proof. □

7. Faculty members

A college has 39 departments, and a total of 262 faculty members in those departments. Prove that there are three departments in this college that have a total of at least 21 faculty members.

Click here for a hint:

Proof. □

8. Triples

Find all triples of positive integers $x < y < z$ for which

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$$

holds.

Proof. □

9. 4-tuples

Find all quadruples (a, b, c, d) of distinct positive integers so that $a < b < c < d$ and
$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} = 1.$$
Click here for a hint:

Proof. □

10. Write Your Own Pigeon-Hole Principle Problem

Please write your own problem and solve it using the Pigeon-hole Principle. (Student's problems may be chosen for future exams' questions).

11. Miscellaneous

(1) Share your work (at least one problem) and thought process with at least one classmate. Ask them to share their thought process as well. Write down their names and briefly summarize your interactions. A virtual discussion via Piazza or email is fine if you don't have time to interact in person.
(2) Approximately how much time did you spend on this homework?