Exam 1 Review Questions - Calculus III - Spring 2024

Last edited: Feb 14 at 4:40pm

Exam 1 is on Friday, Feb 16 in class and covers sections 12.1, 13.1-13.6, 14.1-14.5 (except 14.3), 15.1.

MML Review

To review MML homework, go to your "Gradebook" on Pearson, select "entire course to date" on the drop-down menu, then click on "Review".

1. MML Section 13.1 Problems 1, 2, 3, 4, 6, 10, 13, 15
2. MML Section 13.2 Problems 2, 3, 4, 6, 7, 11, 12, 13
3. MML Section 13.3 Problems 1, 2, 3, 4, 5, 6–8 (leave the angle as the arccosine of a number), 9
4. MML Section 13.4 Problems 1–15
5. MML Section 12.1 Problems 2, 4, 6
6. MML Section 13.5 Problems 1–5, 7, 10–12
7. MML Section 13.6 Problems 1, 2, 3, 4, 8, 9, 10
8. MML Section 14.1 Problems 1, 3, 4, 6, 7, 8, 9
9. MML Section 14.2 Problems 3, 4, 5, 8, 11, 12
10. MML Section 14.4 Problems 1, 2, 3, 4, 6, 8, 10
11. MML Section 14.5 Problems 1–8 (all)
12. MML Section 15.1 Problems 1, 3, 4, 6, 7

Additional review problems

1. Consider the plane R which contains the line $\mathbf{r}(t) = \langle 1+3t, 2t, 7-5t \rangle$ and the point $P(3,1,4)$.
   (a) Find a normal vector to the plane R.
   (b) Find an equation for the plane R.
2. Consider the line ℓ in $\mathbb{R}^3$ which is parallel to the vector $\mathbf{v} = \langle 2, -2, 1 \rangle$ and which contains the point $P(5,0,-3)$. Also, let Q be the point $(6,3,2)$, and let $\mathbf{u} = \overrightarrow{PQ}$.
   (a) Find the equation of the plane which is parallel to the yz-plane and contains the point P.
   (b) Compute the components of the vector $\mathbf{u}$. Express your answer in the form $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$.
   (c) Find an equation for the sphere S which contains the point Q and has center P.
   (d) Write inequalities representing each of the following sets of points:

i. the set of points outside of the sphere S from part (a)

ii. the set of points inside the sphere S, including the points on the sphere itself

(e) Find a parametrization of the line ℓ using the parameter t so that the point P corresponds to $t = 0$. Express your answer as a vector function $\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle$.

(f) Use the dot product to set up an expression that would allow you to compute the angle θ between the vectors $\mathbf{u}$ and $\mathbf{v}$. (*Hint:* Your expression should involve an inverse trig function applied to a number.)

(g) Let θ be the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$. Compute $\sin\theta$.

(h) Compute the unit vector in the direction of $\mathbf{v}$.

(i) Compute the vector in the direction opposite to $\mathbf{v}$ of length 7.

(j) Let $\mathbf{p} = \text{proj}_{\mathbf{v}}\mathbf{u} = \frac{2}{9}\mathbf{i} - \frac{2}{9}\mathbf{j} + \frac{1}{9}\mathbf{k}$. Compute the vector $(\mathbf{u} - \mathbf{p})$, then compute the dot product $(\mathbf{u} - \mathbf{p}) \cdot \mathbf{v}$. What is the angle between the vectors $(\mathbf{u} - \mathbf{p})$ and $\mathbf{v}$?

(k) Consider the parallelogram with two sides given by the line segments $\overline{OP}$ and $\overline{OQ}$, where the point O is the origin $(0, 0, 0)$. Compute the area of the parallelogram.

3. Consider the line segment between $P(4, -1, 2)$ and $Q(7, 0, 6)$.

(a) Find a vector function $\mathbf{r}(t)$ for this line segment so that the parameter t satisfies $0 \leq t \leq 1$, the point P corresponds to $t = 0$, and the point Q corresponds to $t = 1$.

(b) Find the speed of the function $\mathbf{r}(t)$.

(c) Compute the arc length function $s(t)$, where $s(a)$ is the length of the curve $\mathbf{r}(t)$ from $t = 0$ to $t = a$.

(d) Find an arc length parametrization of the line segment $\overline{PQ}$ using arc length s as a parameter. Be sure to give a new interval of values for s.

(e) Find the coordinates of a point R on the line segment $\overline{PQ}$ so that the distance from P to R is 3.

4. Let $\mathbf{r}(t) = e^{3t}(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$. Compute the following:

(a) $\displaystyle\int \mathbf{r}(t)\, dt$

(b) $\mathbf{r}'(t)$

5. Compute $\displaystyle\int_0^{2\pi/3} (\sin t\mathbf{i} + 10\cos(t/2)\mathbf{j} + 18t\mathbf{k})\, dt$.

6. Consider the vector function $\mathbf{r}(t) = \langle 2t, t^2, \frac{t^3}{3}\rangle$.

(a) Find the velocity and speed of the function $\mathbf{r}(t)$.

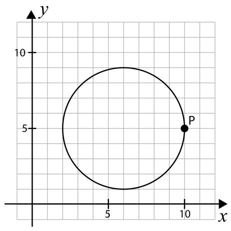
(b) Compute the unit tangent vector $\mathbf{T}(t)$.

(c) Compute the curvature at $t = 1$.

Hint: One of the formulas is much easier to use here!

(d) Find all points of intersection between the curve $\mathbf{r}(t)$ and the plane $4x - 6y + 3z = 0$.

7. Find parametric equations for a path around the circle below with starting point P. The path should start and end at P in a single counterclockwise loop. Be sure to state the range of values for t.



8. Which of the following equations represents a *cylinder* in $\mathbb{R}^3$? For each graph that is a cylinder, state which axis the graph is parallel to. Also, sketch the trace in the appropriate coordinate plane.

(a) $x^2 + \dfrac{z^2}{4} = 1$ (b) $x^2 + y^2 + \dfrac{z^2}{4} = 1$ (c) $y^2 - \dfrac{z^2}{4} = 1$

Answers to additional problems:

1. (a) A vector which is parallel to the line is $\langle 3, 2, -5\rangle$. Pick a point Q on the line, for example $Q(1, 0, 7)$. Your answer could be the cross product $\overrightarrow{QP} \times \langle 3, 2, -5\rangle$ $= \langle 2, 1, -3\rangle \times \langle 3, 2, -5\rangle = \boxed{\langle 1, 1, 1\rangle}$, or any nonzero scalar multiple such as $\boxed{\langle -3, -3, -3\rangle}$.

   (b) $(x-3)+(y-1)+(z-4) = 0$ or $x+y+z = 8$. Equivalently, any nonzero multiple of this equation such as $5x + 5y + 5z = 40$ would work.

2. (a) $x = 5$

   (b) $\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$

   (c) $(x - 5)^2 + y^2 + (z + 3)^2 = 35$

   (d) i. $(x - 5)^2 + y^2 + (z + 3)^2 > 35$
   ii. $(x - 5)^2 + y^2 + (z + 3)^2 \le 35$

   (e) $\mathbf{r}(t) = \langle 5 + 2t, -2t, -3 + t\rangle$

   (f) $\frac{\sqrt{314}}{3}$

   (g) $\frac{\sqrt{314}}{3\sqrt{35}}$
   Note: How would you use the cross-product to compute this?

   (h) $\frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$

   (i) $-\frac{14}{3}\mathbf{i} + \frac{14}{3}\mathbf{j} - \frac{7}{3}\mathbf{k}$

   (j) $\mathbf{u} - \mathbf{p} = \frac{7}{9}\mathbf{i} + \frac{29}{9}\mathbf{j} + \frac{44}{9}\mathbf{k}$
   The dot product is 0 and the angle is $\frac{\pi}{2}$.

   (k) $\sqrt{1090}$

3. (a) $\mathbf{r}(t) = \langle 4 + 3t, -1 + t, 2 + 4t\rangle$

   (b) $|\mathbf{r}'(t)| = \sqrt{26}$

   (c) $s(t) = \sqrt{26}t$

   (d) $\langle 4 + \frac{3}{\sqrt{26}}s, -1 + \frac{1}{\sqrt{26}}s, 2 + \frac{4}{\sqrt{26}}s\rangle$
   $0 \le s \le \sqrt{26}$

   (e) Scalar multiply the unit vector in the direction of $\overrightarrow{PQ}$ by 3 to get a vector of length 3 in the same direction: $\langle \frac{9}{\sqrt{26}}, \frac{3}{\sqrt{26}}, \frac{12}{\sqrt{26}}\rangle$. Then add this length-3 vector to the point P: $\boxed{\langle 4 + \frac{9}{\sqrt{26}}, -1 + \frac{3}{\sqrt{26}}, 2 + \frac{12}{\sqrt{26}}\rangle}$

4. (a) $\dfrac{1}{3}e^{3t}\mathbf{i} + \dfrac{2}{3}e^{3t}\mathbf{j} - e^{3t}\mathbf{k} + \mathbf{C}$
   where $\mathbf{C} = C_1\mathbf{i} + C_2\mathbf{j} + C_3\mathbf{k}$ is an arbitrary constant vector.

   (b) $3e^{3t}\mathbf{i} + 6e^{3t}\mathbf{j} - 9e^{3t}\mathbf{k}$

5. $\frac{3}{2}\mathbf{i} + 10\sqrt{3}\mathbf{j} + 4\pi^2\mathbf{k}$

6. (a) velocity $\mathbf{v}(t) = \langle 2, 2t, t^2\rangle$
   speed $|\mathbf{v}(t)| = t^2 + 2$

   (b) $\langle \frac{2}{t^2+2}, \frac{2t}{t^2+2}, \frac{t^2}{t^2+2}\rangle$

   (c) $2/9$

   (d) $(0, 0, 0), (4, 4, \frac{8}{3}), (8, 16, \frac{64}{3})$

7. $x = 6+4\cos t$, $y = 5+4\sin t$, where $0 \le t \le 2\pi$.

8. (a) is a cylinder parallel to the y-axis. The trace in the xz-plane is an ellipse.

   (b) is not a cylinder.

   (c) is a cylinder parallel to the x-axis. The trace in the yz-plane is a hyperbola.