Math3230 Vocabulary and Reading HW 4

FIRST AND LAST NAME

Submit study notes on the following. Complete by hand or via Overleaf.

1. Write down the definition of a (right) group action. Use the definition in Slides 5.1 or Visual Group Theory (VGT) textbook (Sec 9.1 pg 194-199). Free access to VGT: https://ebookcentral.proquest.com/lib/UCONN/detail.action?docID=3330331

2. Required if you miss class on Monday, but optional for everyone else:
   - Watch M. Macauley's video for slides 5.1.
   - Write down the (right) action of the group D_4 on the set of seven "binary squares" in Slides 5.1. Draw the action diagram.
   - Write down the (right) action of the group D_4 on itself via right multiplication. Draw the action diagram (in this case, it should look exactly like a Cayley diagram for D_4)

3. Take notes of Slides 5.2.
   (a) Write the definition of the *orbit* of $x \in S$ under a group action of G acting on the set S. Is $\text{Orb}(x)$ a subset of S or a subset of G?
   (b) Write down the definition of the *stabilizer* of an object $x \in S$ in G. Is $\text{Stab}(x)$ a subset of S or a subset of G?
   (c) Write down the definition of a fixed point of a group action. The set of all fixed points of a group action is denoted $\text{Fix}(\phi)$. Is $\text{Fix}(\phi)$ a subset of S or a subset of G?
   (d) For example 2, compute all orbits, compute the stabilizer $\text{Stab}(x)$ for all $x \in S$, and compute the set of all fixed points. (First try to compute on your own, the look up answers in Slides 5.2).
   (e) For example 3, write an explanation to yourself that there is no fixed point, so $\text{Fix}(\phi) = \emptyset$. Then compute all orbits and compute the stabilizer $\text{Stab}(x)$ for all $x \in S$. (First try to compute on your own, then look up the answers in *Slides 5.3*, slide no. 3.)
   (f) Prove that $\text{Stab}(x)$ is a subgroup of G for $x \in S$. First attempt this yourself, then watch the video for Slides 5.2 (slide no. 4) - you have to play the video to see the proofs (they are not typed).
   (g)
   - Write down the lemma for the Orbit-Stabilizer theorem (Slides 5.2, Lemma 1)
   - (This is a longer proof) Following Slides 5.2, write a "presentation notes" on the proof of Lemma 1. Divide into parts the proofs that (i) the map is well-defined, (ii) the map is injective, and (iii) the map is surjective, like in the slides.
     Optional: tape yourself explaining this proof on a sheet of paper or a board.

   (h)
   - Write down the Orbit-Stabilizer theorem (Slides 5.2, Theorem 1).
   - (This is much shorter) Following Slides 5.2, use Lemma 1 to prove the Orbit-Stabilizer theorem.

   (i) Any questions?

4. Optional: It might help you understand (right) group action if you learn more about the difference between right group action and left group action.
   - In Example 3 of Slides 5.1, you see the action diagram of the (right) group action of G acting on itself via right multiplication. The action diagram is identical to the Cayley diagram of G.
     Now, try drawing the action diagram of the (left) group action of G acting on itself via left multiplication. The action diagram should look *different* (although similar) to Example 3.
   - Find examples of right vs left group action, go to the Wikipedia page for group action or google this question.