Math3230 Vocabulary and Reading HW 2

FIRST AND LAST NAME

Submit study notes on the following. Complete by hand or via Overleaf.

1. Review the definition of a set partition and equivalence relation again, but this time use group theory to give an example of an equivalence relation. Let H be a subgroup of a group G, and explain to yourself how the set of left cosets defines an equivalence relation on the group G. Copy the explanation from class (Slides 3.1-3.3) or Visual Group Theory (VGT) textbook (Chapter 6). Link to free access to VGT: https://ebookcentral.proquest.com/lib/UCONN/detail.action?docID=3330331

2. Look up the following in Chapter 6 of VGT or Slides 3.1-3.2.
   (a) What to do to check that a subset of a group is a subgroup
   (b) What to do to check that a subset of a group is *not* a subgroup
   (c) Definition of a left coset of a subgroup
   (d) Definition of a right coset of a subgroup
   (e) Copy down Lagrange's theorem
   (f) Definition of the index of a subgroup H in G.

3. Look up the following in Slides 3.3.
   (a) Definition of a normal subgroup (Def 7.2 in VGT or Slides 3.3)
   (b) Definition of a conjugate of a subgroup
   (c) Definition of the center of a group
   (d) Statements Theorem 3 and all propositions in Slides 3.3
   (e) Write down Lagrange's theorem
   (f) Definition of the index of a subgroup H in G.

4. Take study notes for direct products from Slides 3.4 and Group Explorer
   (a) Definition of a direct product
   (b) Reproduce three of the Cayley diagrams of the examples in Slides 3.4. Make sure you write down which Cayley diagram goes with which group.
   (c) Go to the "practice" block on slides no. 7. Go to Group Explorer to look at the 16 subgroups of the group $C_2 \times C_2 \times C_2$, then write down here all subgroups which
   (d) Go to the database of small groups in Group Explorer: nathancarter.github.io/group-explorer/GroupExplorer.html. For the non-prime integers $n = 4$, 6, 8, 9, 10, 12, 14, 15, 16, 18, and 20, write down several cases where $\mathbb{Z}_n$ is NOT ISOMORPHIC to the product $\mathbb{Z}_a \times \mathbb{Z}_b$ where $ab = n$. This does not need to be a complete list.

      To tell whether $\mathbb{Z}_n$ is NOT ISOMORPHIC to $\mathbb{Z}_a \times \mathbb{Z}_b$, just check that both $\mathbb{Z}_n$ and $\mathbb{Z}_a \times \mathbb{Z}_b$ show up on this website.

      For example, Z_4 and $V_2 = C_2 \times C_2$ both show up on the website, so they are not isomorphic. However, $\mathbb{Z}_6$ shows up but $\mathbb{Z}_2 \times \mathbb{Z}_3$ does not show up, so $\mathbb{Z}_6$ IS isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_3$.
   (e) Based on the examples above, try to form a conjecture about when $\mathbb{Z}_n$ has the same structure as $\mathbb{Z}_a \times \mathbb{Z}_b$. Attempt to prove your conjecture. If you can't, attempt to disprove it.