Math3230 Vocabulary and Reading HW 1

FIRST AND LAST NAME

Write up the following reading homework. They could be completed by hand or via Overleaf.

1. (REVIEW OF TRANSITION TO ADVANCED MATH) Submit about 1-2 pages of notes explaining to another math student the following:

   (a) definition of partition

   (b) definition of equivalence relation

   (c) definition of congruence modulo n

   (d) two to four more examples of an equivalence relation

   The following sources would contain all these information: Your old textbook from Transition to Advance Math, the following section in T. Judson's online textbook:
   s abstract.ups.edu/aata/section-sets-and-equivalence-relations.html#VFe, or other intro-to-proofs textbooks.

2. (REVIEW OF GROUP) Submit study notes with the following definition.

   (a) What does it mean for a binary operation to be *associative*?

   (b) Suppose S is a set with a binary operation $\circ$. What does it mean for an element of S to be an *identity* element for the binary operation $\circ$?

   (c) Suppose Suppose S is a set with a binary operation $\circ$, and suppose an identity element exists. Let x be an element of S. What does it mean for a element of S to be an *inverse* of x?

   (d) Write down the definition of a group (using the language of set and binary operation).

   (e) Let G be a group. Write down the definition of the *order of* G.

   (f) Let x be an element in a group G. What is the definition of the *orbit of* x?

   (g) Let x be an element in a group G. What is the definition of the *order* of x?

   (h) Go to the database of small groups in Group Explorer: nathancarter.github.io/group-explorer/GroupExplorer.html. Check out the following eleven groups: the first nine groups on this list (that is, all groups with order 7 or less), and two groups of order 8 called D_4 and Q_4. Copy down the information for the group presentation, Cayley diagram, object of symmetry (if any) and cycle graph for each of these 11 groups.

   (i) We have seen all 11 groups during class. For each of these groups, write a note connecting it to how it was described in class (for example, "cyclic group of order 5", "Klein 4", "lightswitch", "rectangle puzzle", "abelian group", "hexagon symmetry", "Dihedral group of order 8", etc).

   (j) Note that there are two different groups of order 4 and also two different groups of order 6. Pick either $n = 4$ and $n = 6$. Observe the patterns of the multiplication tables of the two groups of order n. Write down an instruction to your future self how you would be able to tell which group of order n is which just by the pattern of the multiplication table, just by the object of symmetry, and just by the cycle graph.