Math3230 Abstract Algebra Homework 6 (Team Writeup)

- Each team should submit one copy of the sections that the team is assigned to.
- In this writeup, your proofs should be more detailed than the proofs given in Judson and slides/videos. You can either submit a hand-written writeup in class or share an Overleaf project.
- Note: *factor groups* are the same as *quotient groups*.

1 Fundamental homomorphism theorem (First Isomorphism Theorem)

Use Slides 4.3 and M. Macauley's video of Slides 4.3, and proof of Theorem 11.10 of Judson: abstract.ups.edu/aata/section-group-isomorphism-theorems.html as resources.

Write down the statement of the Fundamental Homomorphism Theorem (First Isomorphism Theorem)

Write the definition of the map i which is the bijective homomorphism in the proof.

Prove that i is well-defined

1. Prove that i is a homomorphism. Make sure to remind the audience of the binary operation for a quotient group.

2. Prove that i is onto

3. Prove that i is injective.

4. Example: Define a surjective homomorphism f from $\mathbb{Z}$ to $\mathbb{Z}_n$ in such a way that you can apply the First Isomorphism Theorem to conclude that $\mathbb{Z}/n\mathbb{Z} \cong \mathbb{Z}_n$.

2 Diamond Isomorphism Theorem (Second Isomorphism Theorem)

Use Slides 4.5, M. Macauley's video of Slides 4.5, and 11.12 of Judson: abstract.ups.edu/aata/section-group-isomorphism-theorems.html as resources.

Write down the statement of the Diamond Isomorphism Theorem (Second Isomorphism Theorem).

Draw the "diamond" lattice picture.

Write down the definition of the map ϕ which is the surjective homomorphism in the proof of the theorem.

1. Prove that HN is a subgroup of G.

2. Prove that the intersection $H \cap N$ is a *normal* subgroup of H.

3. Prove that the map ϕ is a homomorphism. Make sure to remind the audience of the binary operation for a quotient group.

4. Prove that the map ϕ is surjective.

5. Prove that the kernel of the map ϕ is equal to $H \cap N$.

6. Apply the Fundamental theorem (First Isomorphism Theorem) to prove the Second Isomorphism Theorem.

3 The Third Isomorphism Theorem

Use Slides **4.5** and M. Macauley's video of Slides **4.5**, and proof of Theorem 11.3 and 11.4 of Judson: abstract.ups.edu/aata/section-group-isomorphism-theorems.html as resources.

Write down the statement of the Third Isomorphism Theorem.

Write down an explanation (to yourself) the picture given in the slides/video.

Write down the definition of the map φ which is the surjective homomorphism in the proof of the theorem.

1. Prove the first part of the theorem, that $H/N \lhd G/N$.

2. Prove that φ is well-defined.

3. Prove that φ is surjective.

4. Prove that φ is a homomorphism.

5. Write down the kernel of φ (and prove your answer).

6. Apply the Fundamental theorem (First Isomorphism Theorem) to prove the Third Isomorphism Theorem