Math3230 Abstract Algebra Homework 6 (Team presentation)

- Prepare a presentation of the two sections your team is assigned to. You will be asked to present just one of the sections. Try to arrange it so that each team member gets to speak roughly the same amount of minutes.
- You should probably use notes (your team writeup) during the presentation.
- During your presentation, your proofs should be a lot more detailed than the proofs given in Judson and slides/videos.
- Note: *factor groups* are the same as *quotient groups*.

1 Fundamental homomorphism theorem (First Isomorphism Theorem)

Use Slides 4.3 and M. Macauley's video of Slides 4.3, and proof of Theorem 11.10 of Judson: abstract.ups.edu/aata/section-group-isomorphism-theorems.html as resources.

Write down the statement of the Fundamental Homomorphism Theorem (First Isomorphism Theorem)

Write the definition of the map i which is the bijective homomorphism in the proof.

Prove that i is well-defined

1. Prove that i is a homomorphism. Make sure to remind the audience of the binary operation for a quotient group.
2. Prove that i is onto
3. Prove that i is injective.
4. Example: Define a surjective homomorphism f from $\mathbb{Z}$ to $\mathbb{Z}_n$ in such a way that you can apply the First Isomorphism Theorem to conclude that $\mathbb{Z}/n\mathbb{Z} \cong \mathbb{Z}_n$.

2 Diamond Isomorphism Theorem (Second Isomorphism Theorem)

Use Slides 4.5, M. Macauley's video of Slides 4.5, and 11.12 of Judson: abstract.ups.edu/aata/section-group-isomorphism-theorems.html as resources.

Write down the statement of the Diamond Isomorphism Theorem (Second Isomorphism Theorem).

Draw the "diamond" picture.

Write down the definition of the map ϕ which is the surjective homomorphism in the proof of the theorem.

1. Prove that HN is a subgroup of G.
2. Prove that the intersection $H \cap N$ is a *normal* subgroup of H.
3. Prove that the map ϕ is a homomorphism. Make sure to remind the audience of the binary operation for a quotient group.
4. Prove that the map ϕ is surjective.
5. Prove that the kernel of the map ϕ is equal to $H \cap N$.
6. Apply the Fundamental theorem (First Isomorphism Theorem) to prove the Second Isomorphism Theorem.

3 The Third Isomorphism Theorem

Use Slides **4.5** and M. Macauley's video of Slides **4.5**, and proof of Theorem 11.3 and 11.4 of Judson: abstract.ups.edu/aata/section-group-isomorphism-theorems.html as resources.

Write down the statement of the Third Isomorphism Theorem.

Write down an explanation (to yourself) the picture given in the slides/video.

Write down the definition of the map φ which is the surjective homomorphism in the proof of the theorem.

1. Prove the first part of the theorem, that $H/N \lhd G/N$.
2. Prove that φ is well-defined.
3. Prove that φ is surjective.
4. Prove that φ is a homomorphism.
5. Write down the kernel of φ (and prove your answer).
6. Apply the Fundamental theorem (First Isomorphism Theorem) to prove the Third Isomorphism Theorem