Math3230 Abstract Algebra Homework 1

your preferred first and last name

0. Read and take notes of the following.

- Section 2.2 – 2.6 of *Visual Group Theory* (VGT), which you can download for free with UConn login: https://ebookcentral.proquest.com/lib/UCONN/detail.action?docID=3330331
- VGT Exercises 2.8, 2.13, 2.15, and 2.17 (including the answers in the back of the book).
- The article *Group Think* by Steven Strogatz, which appeared in the NY Times in 2010: opinionator.blogs.nytimes.com/2010/05/02/group-think/

Write up solutions to the following exercises. Question 1 should be submitted via Overleaf. The rest of the questions could be completed by hand or via Overleaf.

1. Read Francis Su's essay on good mathematical writing: egunawan.github.io/algebra/reading/someguidelines.pdf

   Answer the following questions after reading Francis Su's essay on good mathematical writing.

   (a) What is a good rule of thumb for what you should assume of your audience as you write your homework sets?

   (b) Is whiteboard writing formal or informal writing?

   (c) Why is the proof by contradiction on the 3rd page not really a proof by contradiction?

   (d) Name three things a lazy writer would do that a thoughtful writer would not.

   (e) What's the difference in meaning between these three phrases?

   "Let $A = 12$." "So $A = 12$." "$A = 12$."

2. Given a regular n-gon, let r be a rotation of it by $2\pi/n$ radians. This time, assume that we are not allowed to flip over the n-gon. These n actions form a group denoted $C_n = \langle r \rangle = \{e, r, r^2, \ldots, r^{n-1}\}$.

   (a) Draw a Cayley diagram for C_n for $n = 4$, $n = 5$, and $n = 6$. [*Hint:* You've seen the Cayley diagram for C_4 in class.]

   (b) For $n = 4, 5, 6$, find all minimal generating sets of C_n. [*Hint:* There are minimal generating sets of C_6 of size 2.]

   (c) Make a conjecture of what integers k does $C_n = \langle r^k \rangle$ for a general fixed integer n.

3. As we saw in lecture, the six symmetries of an equilateral triangle 

form a group denoted $D_3 = \{e, r, r^2, f, rf, r^2f\}$, where r is a 120° clockwise rotation and f is a flip about a vertical axis (which fixes the top corner). Since r and f suffice to generate all six of these symmetries, we write $D_3 = \langle r, f \rangle$.

   [*Hint*: Use the Cayley diagram given in Sec 1.1 of the slides. Remember that we read from left to right.]

   (a) Let g be the reflection of the triangle that fixes the lower-left corner. Which of the six actions in D_3 is g equal to? Which action is fg?

   (b) Write all 6 actions of D_3 using only f and g. Draw a Cayley diagram using f and g as generators. [Hint: First write all eight actions of D_3 in terms of f and g. This Cayley diagram should look very different from the Cayley diagram using r and f as generators.]

   (c) To generate D_3, we need at least 2 actions. It is possible to show that if we have 3 generators, then one of them is unnecessary. Find all *minimal* generating sets of $D_3 = \{e, r, r^2, f, rf, r^2f\}$; note that all of them should have exactly two actions. Do not use g in this list.

4. The eight symmetries of a square 

form a group denoted D_4. Let r be a 90° clockwise rotation and f a horizontal flip (that is, about a vertical axis). It is possible to show that $D_4 = \langle r, f \rangle$.

   (a) Write all 8 actions of D_4 using r and f, and draw a Cayley diagram using these two actions as generators. [Hint: Follow the example for D_3 in Sec 1.1 of the slides. One of the exercises from the reading shows you a possible way to arrange the Cayley diagram for D_4]

   (b) Let g be the reflection of the square that fixes the lower-left and upper-right corner. Which of the eight actions in D_4 is g equal to? Which action is fg?

   (c) Draw a Cayley diagram of D_4 using f and g as generators. [Hint: First write all eight actions of D_4 in terms of f and g. This Cayley diagram should look very different from your diagram in part 4a]

   (d) Find all *minimal* generating sets of D_4. [*Hint*: There are 12. Some minimal generating sets like $\langle f, g \rangle$ consist of two flips, and some minimal generating sets like $\langle f, r \rangle$ consist of one flip and one rotation.]

   (e) Find a minimal generating set of D_6 (the 12 symmetries of a regular hexagon) that has three actions.

5. Share your work (at least one bullet point) and thought process with at least one classmate. Ask them to share their thought process as well. Write down their names and briefly summarize your interactions. A virtual discussion via Piazza or email is fine if you were not able to attend class.

6. Credit: Write down everyone who helped you, including classmates who contributed to your thought process. Write down all written sources you used as well.

7. Approximately how much time did you spend on this homework?

Note: See the updated version of Section 1.1 of the slides for some hints. You are welcome to ask for further hints during class, drop-in hour, or MWF office hour (by appointment). You can also post and answer questions on Piazza (anonymously if you prefer).