You may rip out this sheet and use it during the test.

Fact Sheet

Definition 1. The *order* of a group element x is the size of its orbit $\langle x \rangle$. Note: If the size of $\langle x \rangle$ is finite, then the order of x is the smallest positive integer k such that $x^k = e$.

Definition 2. The *order* of a group G is the number of elements in G.

Theorem 3. If a permutation σ can be expressed as the product of an even number of transpositions, then any other product of transpositions equaling σ must also contain an even number of transpositions. Similarly, if σ can be expressed as the product of an odd number of transpositions, then any other product of transpositions equaling σ must also contain an odd number of transpositions.

Remark 4. Suppose J is a subset of a group G. To show that J is a subgroup of G, you need to check the following.

(a) J contains the identity of G

(b) for all $x, y \in J$, the product xy is also in J

(c) for all $x \in J$, the inverse x^{-1} is also in J

Theorem 5. Assume G is finite. If H is a subroup of G, then $|H|$ divides $|G|$.

Proof. Suppose there are n left cosets of the subgroup H. Since they are all the same size and they partition G, we must have $|G| = \underbrace{|H| + \cdots + |H|}_{n \text{ copies}} = n\,|H|$. $\square$

Corollary 6. If G is a finite group and H is a subgroup of G, then $[G : H] = \frac{|G|}{|H|}$.