1 Some cyclic groups

Let $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix}$, and $\sigma = (126)(45) = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 6 & 3 & 5 & 4 & 1 & 7 \end{bmatrix} \in S_7$

1.1 For each group below, if the group is finite, list all elements; if the group is infinite, describe all elements

1. $\langle$2D rotation by $240^o\rangle$ 2. $\langle\sigma\rangle$ 3. $\langle A\rangle$ 4. $\langle B\rangle$ 5. $\langle AB\rangle$

1.2 Match the groups from Question 1.1 with the groups below

(i) $\mathbb{Z}_2$ ____ (ii) $\mathbb{Z}_3$ ____ (iii) $\mathbb{Z}_4$ ____ (iv) $\mathbb{Z}_6$ ____ (v) $\mathbb{Z}$ ____

2 Twenty four

(a) If a group element x is such that $x^{24} = e$, what are the possible orders of x?
(b) Draw the subgroup lattice of $\mathbb{Z}_{24}$

3 Book exercises

Gallian Chapter 4 Exercise 1 (pg 85); Exercise 3 (pg 86); Exercise 19 (pg 87); Exercise 21 (pg 87)

4 A cycle of odd length

- Warm-up: First compute $(15428)^2 = (15428)(15428) = ...$

- Suppose σ is a cycle of odd length $(a_1\ a_2 \ldots\ a_{2k}\ a_{2k+1})$. Compute σ^2.

- If σ is a cycle of odd length, prove that σ^2 is also a cycle.

5 Product of transpositions (lemma)

a. Example: First try writing (25) as a finite product of $(12), (13), (14), \ldots, (1n)$.
(An answer is given at the end of this document)

Warm-up: Now write (37) as a finite product of $(12), (13), (14), \ldots, (1n)$.

Write the transposition $(a\,b)$ as a finite product of $(12), (13), (14), \ldots, (1n)$.

b. Example: First try writing (25) as a finite product of $(12), (23), (34), \ldots, (n-1, n)$.
(An answer is given at the end of this document.)

Warm-up: Now write (15) as a finite product of $(12), (23), (34), \ldots, (n-1, n)$.

Write the transposition $(a\,b)$ as a finite product of $(12), (23), (34), \ldots, (n-1, n)$.

6 Theorems

a. Prove that any permutation in S_n can be written as a finite product of $(12), (13), (14), \ldots, (1n)$.

b. Prove that any permutation in S_n is a finite product of $(12), (23), (34), \ldots, (n-1, n)$.

Warm-up: $(25) = (12)(15)(12)$
Warm-up: $(25) = (23)(34)(45)(34)(23)$