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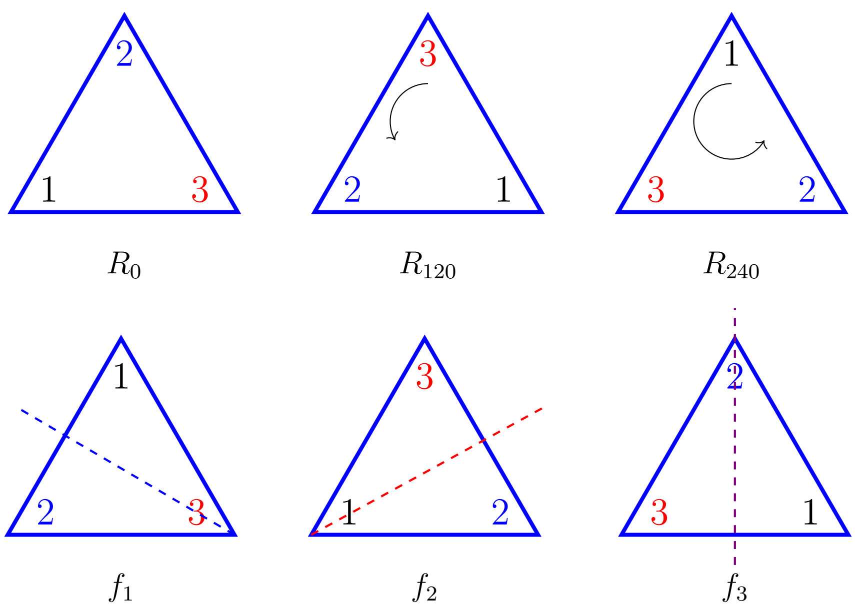
Group D_3 of symmetries of a triangle

A **symmetry** or **rigid motion** of a figure in the plane $\mathbb{R}^2$ is a a rearrangement of the figure preserving the arrangement of its sides and vertices as well as its distances and angles.

It is the final position of the figure that is important, not the motion itself. For example, if we rotate the triangle below through 120° or 480° the triangle ends up in the same final position, so we do not think of these as distinct symmetries.

Consider the rigid motions of an equilateral triangle, as illustrated below. Each sketch shows the resulting position when the specified motion is applied to the triangle starting in the original position.



We can apply one of the rigid motions, and then, *continuing from the new position of the triangle*, apply another of the rigid motions. We can then record the overall effect as one of six symmetries listed above.

For example, if we first apply f_1, then (continuing from the resulting position) apply R_{120} (a 120° counterclockwise rotation) we end up with f_2. Using our usual function composition notation, we can think of this as $R_{120} \circ f_1$. In fact, we write $R_{120} \circ f_1 = f_2$.

It is useful to write the results of all such combinations in a **Cayley table**, as shown below. For example, $R_{120} \circ f_1 = f_2$ is written in the row labeled R_{120} and the column labeled f_1.

1.

$\circ$	R_0	R_{120}	R_{240}	f_1	f_2	f_3
R_0						
R_{120}				f_2		
R_{240}			R_{120}			
f_1		f_3			R_{240}	R_{120}
f_2						
f_3				R_{240}	R_{120}	R_0

Use the cut-out triangle to determine the compositions and use this to complete the table. We will denote this collection of symmetries, together with composition, by D_3.

2. Give an example from the table that shows that the order in which you apply the rigid motions matters. Describe this both using our notation and geometrically.

3. There there are examples from the table for which the order doesn't matter. List some.

4. Does R_{120} have an inverse, i.e. is there another motion we can compose it with to get back to the original position? If so, describe it both using our notation and geometrically.

5. Explain geometrically why a reflection (or flip) followed by a reflection must be a rotation.

6. Explain geometrically why a rotation and a reflection/flip taken together in either order must be a reflection.