For Exam 1, if relevant to the questions, the following will be printed on the same page as the questions.

Definition 1. The *order* of a group element x, denoted by $|x|$, is the size of its orbit $\langle x \rangle$. Note: If the size of $\langle x \rangle$ is finite, then the order of x is the smallest positive integer k such that $x^k = e$.
The *order* of a group G, denoted by $|G|$, is the number of elements in G.

Remark 2. Let J be a subset of a group G. To show that J is a subgroup of G, show the following:

(a) J contains the identity of G

(b) for all $x, y \in J$, the product xy is also in J (closure under the group operation)

(c) for all $x \in J$, the inverse x^{-1} is also in J (closure under taking inverses)

Theorem 3. If a permutation σ can be expressed as the product of an even number of transpositions, then any other product of transpositions equaling σ must also contain an even number of transpositions. Similarly, if σ can be expressed as the product of an odd number of transpositions, then any other product of transpositions equaling σ must also contain an odd number of transpositions.

Proposition 4. For any $\sigma \in S_n$, we have $\sigma \; (a_1 \; a_2 \; \ldots \; a_k) \; \sigma^{-1} = (\, \sigma(a_1) \; \sigma(a_2) \; \ldots \; \sigma(a_k) \,)$.

Definition 5. A group *homomorphism* is a function $\phi\colon (G_1, *) \to (G_2, \circ)$ satisfying

$$\phi(a * b) = \phi(a) \circ \phi(b), \qquad \text{for all } a, b \in G_1 \,.$$

Proposition 6. Let $f : G_1 \to G_2$ be a homomorphism of groups. If e_1 is the identity of G_1, then $f(e_1)$ is the identity of G_2.