First & Last Name: _____ Student ID: _____

Abstract Algebra Individual Quiz 6

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1. Consider the homomorphism $f : \mathbb{Z} \to \mathbb{C}^*$ defined by

$$f(m) = i^m$$

(a) What is the kernel of f?

Solution: $4\mathbb{Z} = \{4m : m \in \mathbb{Z}\} = \{..., -8, -4, 0, 4, 8, ...\}$

(b) Is f injective? Why or why not?

Solution: No. For example, $f(4) = i^4 = 1 = i^0 = f(0)$.

(c) What is the image of f?

Solution: $\{1, i, -1, -i\} = \{1, e^{i(\pi/2)}, e^{i(\pi/2)2}, e^{i(\pi/2)3}\}$, the 4-th roots of unity.

(d) Is f an isomorphism?

(yes/ no)

Solution: No, f is not an isomorphism because f is not a bijection.

2. Let $H = \{Id,\ (12)(35),\ (13)(25),\ (15)(23)\}$, a subgroup of S_5.

Prove or disprove: there is an isomorphism from $\mathbb{Z}_4 = \{0, 1, 2, 3\}$ to H.

Solution: False.
We have $x^2 = Id$ for all $x \in H$, so no element of H can generate the entire group H. This means H is not a cyclic group.

3. (I) Fill in the table so that it depicts the Cayley table of a group $G = \{e, a, b\}$, with e as the identity element. There may be more than one way to complete a table, and if so you need to give all possibilities.

(Note: Many copies of this table are printed below so that you can use them for your scratch work.)

	e	a	b
e			
a			
b			

	e	a	b
e			
a			
b			

	e	a	b
e			
a			
b			

	e	a	b
e			
a			
b			

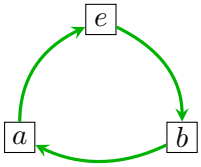
Solution: There is only possibility

(II) Circle one of the tables you have completed. Write down a minimal generating set for the group corresponding to this table.

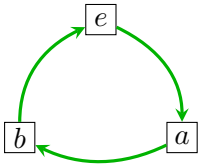
Solution: There are only two possible minimal generating sets, either $\{a\}$ or $\{b\}$.

(III) Draw the Cayley diagram for the minimal generating set that you wrote above.

Solution: If you choose your minimal generating set to be $\{b\}$, then your Cayley diagram would be the following (with each arrow labeled b):



If you choose your minimal generating set to be $\{a\}$, then your Cayley diagram would be the following (with each arrow labeled a):



(IV) What is the order of the element b in the group whose Cayley table you circled above?

Solution: 3