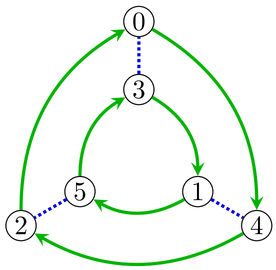
1. The group $(\mathbb{Z}_6, +)$ has a minimal generating set $S = \{3, 4\}$. Draw the Cayley graph with this S as the generating set.

Solution: Each solid (green) arrow has label $+4$. Each dotted (blue) edge has label $+3$.



Note: To distinguish the two types of arrows, label them by $+3$ and $+4$.

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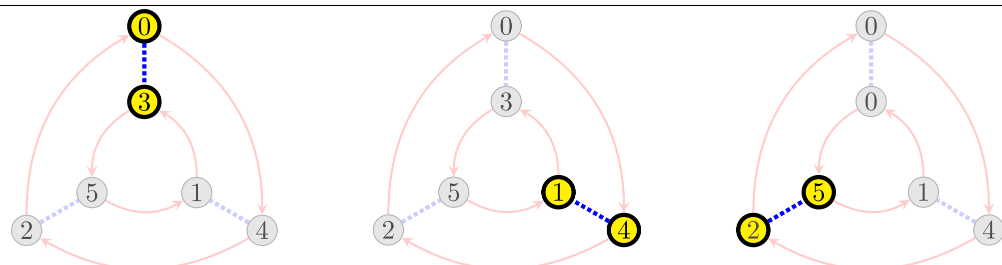
2. Let $H = \langle 3 \rangle = \{0, 3\}$ denote the cyclic subgroup of $\mathbb{Z}_6$ generated by 3. Write down all the left cosets of H.

Solution:



Answer: The left (also right) cosets of H are

- $H = \{0, 3\}$
- $1 + H = \{1, 4\}$
- $2 + H = \{2, 5\}$

Note: Recall that a left coset of H in $\mathbb{Z}_6$ is the set $g + H = \{g + h : h \in H\}$.

Note: You can use the Cayley graph from above question to help you, but you don't have to.

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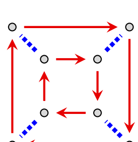
3. What is the index of $H = \{0, 3\}$ in $\mathbb{Z}_6$? (fill in the correct square)

Solution: 3

Note: Recall that the index $[\mathbb{Z}_6 : H]$ is the number of left cosets of H in $\mathbb{Z}_6$.

4. Circle the Cayley graph below which is the correct graph for the direct product $\mathbb{Z}_2 \times \mathbb{Z}_4$ with generating set consisting of the elements $(1, 0)$ and $(0, 3)$.

Solution: 

If you finish the quiz early, work on the following problem. (This page will not be graded.)

Let G denote the direct product $\mathbb{Z}_3 \times \mathbb{Z}_2$.

(a.) List all elements of the orbit of the element $(1, 1)$ in G. Recall that the orbit of an element x is the cyclic subgroup generated by x.

Solution: The orbit of $(1, 1)$ is in fact the entire $\mathbb{Z}_3 \times \mathbb{Z}_2$.

(b.) Draw the Cayley diagram of $\langle (1, 1) \rangle$ using $(1, 1)$ as a generator.

Solution: Let the arrow be labeled by the generator $(1, 1)$.