First & Last Name: _____ Student ID: _____

Abstract Algebra Individual Quiz 4

---

1. Write $(3\,7)$ as a finite product of transpositions from the following list: $(1\,2), (2\,3), (3\,4), (4\,5), (5\,6), (6\,7)$.

Solution: $(3\,7) = (3\,4)(4\,5)(5\,6)(6\,7)(5\,6)(4\,5)(3\,4)$

---

2. Write down all permutations in the alternating group A_3.

Solution:

- Id
- (123)
- (321)

---

3. What is the index of A_3 in S_3? (fill in the correct square)

☐ 1 ☐ 2 ☐ 3! ☐ 4! ☐ ∞

Solution: By Lagrange's theorem, the index $[S_3 : A_3]$ is $\frac{|S_3|}{|A_3|} = \frac{3!}{3} = 2$

---

4. Let G be a group, let H be a subgroup of G, and let $a, b \in G$. Prove that if $a^{-1}b \in H$ then $b \in aH$.

Solution: Suppose $a^{-1}b \in H$. Then $a^{-1}b = h$ for some $h \in H$. This means $b = ah \in aH$.

If you finish the quiz early, work on the following problem. It will come up in the future. (This page will not be graded)

Let G be a group, let H be a subgroup of G, and $g \in G$. Prove the following:

$$\text{If } gH = Hg \text{ then } ghg^{-1} \in H \text{ for all } h \in H.$$

Solution: Suppose $gH = Hg$.

Let $h \in H$. Then $gh \in gH$ by definition.

Since $gH = Hg$ by assumption, $gh \in Hg$. So $gh = kg$ for some $k \in H$. Then $ghg^{-1} = k \in H$.

Give an example of a subgroup H of S_3 and $g \in S_3$ such that gH is not equal to Hg.

Compute the index $[S_3 : H]$ for your chosen H.