

**Abstract Algebra Individual Quiz 2**

Recall that  $U(5)$  is the group of units in  $\mathbb{Z}_5$  with multiplication module 5 as binary operation, that is,

$$U(5) = \{x \in \mathbb{Z}_5 : x \text{ and } 5 \text{ are relatively prime, meaning } \gcd(x, 5) = 1\}$$

**1 Complete the Cayley table for the group  $U(5)$ .**

·	1	2	3	4
1			3	
2			1	
3			4	
4			2	

**Solution:**

·	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

**2 Compute the order of each element in the group  $U(5)$ .**

- 1.
- 2.
- 3.
- 4.

**Solution:**  
 The order of each element:  $|1| = 1$        $|2| = 4$        $|3| = 4$        $|4| = 2$

### 3 Groups of order 8

Consider the group  $\mathbb{Z}_8$  (with addition modulo 8 as group operation), and the “square mattress group”  $D_4$  which is the symmetry group of the regular 4-gon.

Explain why  $\mathbb{Z}_8$  and  $D_4$  do not have the same group structure.

**Solution:**

**A possible explanation:** The group  $\mathbb{Z}_8$  is abelian because addition modulo 8 is commutative. The group  $D_4$  is not abelian because, for example, rotation by  $90^\circ$  does not commute with any of the flips.

**Another possible explanation:** The group  $\mathbb{Z}_8$  has only one element of order two, the element 4. The group  $D_4$  has five elements of order two: all four flips, and the rotation by  $180^\circ$ .