1 Question

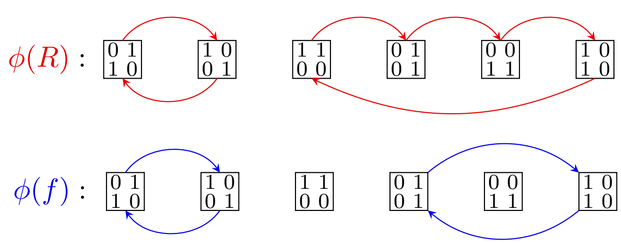
Let G be a group and let $a_1, a_2, \ldots, a_k \in G$. What does it mean to say that G is generated by $a_1, a_2, \ldots, a_k$?

2 Question

Consider the group $G = D_4$. Let R be the counterclockwise rotation by 90^o, and f the horizontal flip (that is, the reflection with respect to a vertical mirror). Then our G can be generated by R and f.

$$\text{Let} \quad X = \left\{ \boxed{\begin{matrix}0&1\\1&0\end{matrix}}\ , \boxed{\begin{matrix}1&0\\0&1\end{matrix}}\ , \boxed{\begin{matrix}1&1\\0&0\end{matrix}}\ , \boxed{\begin{matrix}0&1\\0&1\end{matrix}}\ , \boxed{\begin{matrix}0&0\\1&1\end{matrix}}\ , \boxed{\begin{matrix}1&0\\1&0\end{matrix}} \right\}$$

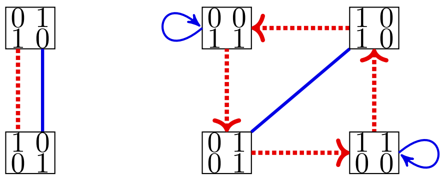
Let $\phi : D_4 \to \mathrm{Perm}(X)$ be the group homomorphism which sends each motion $g \in D_4$ to the permutation of X resulted from applying g to the all squares. For example, below are visual representations of the permutations $\phi(R)$ and $\phi(f)$:



Draw the permutation $\phi(Rf)$ using the same "cycle notation". Recall that we read from right to left, so Rf is the result of applying the flip f and then the rotation R.

3 Question

The following is the action diagram (of the previous group action) using the generators R, f of D_4:



(a) Draw the action diagram using the generators f and g of D_4, where g is the diagonal flip which swaps the northeast and southwest corners. (Hint: is it true that $g = Rf$?)

(b) For shorthand, think of the squares in X as $1, 2, 3, 4, 5, 6$. Now we can write $\phi(R) = (12)(3456)$ and $\phi(f) = (12)(46)$ and $\phi(Rf) = (34)(56)$. Write $\phi(g)$ in this cycle notation for the other 5 elements $g \in D_4$.

(c) What familiar group is $\phi(G)$?

4 Question

(a) Finish calculating the left regular representation of $U(12) = \{1, 5, 7, 11\}$.

- T_7

- T_{11}

(b) What familiar abelian group is $U(12)$?

5 Question (Linear Algebra Example)

Consider the example of how $GL_2(\mathbb{R})$ acts on $X = \mathbb{R}^2$ by left multiplication. Describe this same action using the alternative definition

$$\phi : GL_2(\mathbb{R}) \to \mathrm{Perm}(\mathbb{R}^2).$$