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1 Computation

1.1 Arithmetic

Suppose a, b, c are elements of a group G, and suppose $|a| = 6$ and $|b| = 7$. Express

$$(a^4c^{-2}b^4)^{-1}a^3$$

without using negative exponents.
(Hint: Use "socks-shoes" property for inverses, laws of exponents, and the order of a and b)

1.2 If a group element x is such that $x^{24} = e$, what are the possible orders of x?

1.3 Draw the subgroup lattice of $\mathbb{Z}_{24}$.

Let $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix}$, and $\sigma = (126)(45) = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 6 & 3 & 5 & 4 & 1 & 7 \end{bmatrix} \in S_7$

2 For each group below, if the group is finite, list all elements; if the group is infinite, describe all elements

1. $\langle$2D rotation by $240^o\rangle$

2. $\langle A \rangle$

3. $\langle B \rangle$

4. $\langle AB \rangle$

5. $\langle \sigma \rangle$

6. The symmetry group of a (non-square) rectangle, aka the "mattress group" from the first day

3 Match the groups from Question 2 with the groups below

(i) $\mathbb{Z}_2$ _____ (ii) $\mathbb{Z}_3$ _____ (iii) $\mathbb{Z}_4$ _____ (iv) $\mathbb{Z}_6$ _____ (v) $\mathbb{Z}$ _____ (vi) $\mathbb{Z}_2 \times \mathbb{Z}_2$ _____

4 True or false

Let G denote a finite group. If the statement is true, give a proof. If it is false, give a counterexample.

4.1 For all elements $a, b \in G$, we have $|ab| = |ba|$

4.2 For all elements $a, b \in G$, we have $|a| = |b^{-1}ab|$

4.3 For all elements $a, b \in G$, we have $|b| = |b^{-1}ab|$

5 Union of subgroups

5.1 Prove that it is impossible for a group G to be the union of two proper subgroups.

(In your proof, don't assume that the group is finite)

5.2 Let G denote the rectangle (non-square) "mattress group" (the symmetry group of a non-square rectangle). Write G as the union of three proper subgroups.

6 Subgroup

Find all subgroups of the symmetry group of the regular triangle (the "triangle mattress" group).